

Math 216 - Quiz 5

October 6, 2010

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (6 points) Consider the following Bernoulli equation and its initial condition.

$$\frac{dy}{dx} = \frac{y}{2x} - \frac{x}{2y^2}, \quad y(1) = 1$$

- (a) Solve the initial value problem.

- (b) Visit the website <http://www.dartmouth.edu/~rewn/javaindex.html>. Use the DIRFLD applet to sketch the direction field for the equation. Move the cursor to the point (1, 1) and click in order to sketch the solution curve for the given initial condition. Based on the direction field and the solution curve, explain why it is probably not likely that a numerical solution method will produce good results if you start at (1, 1).

- (c) With $h = 0.1$, use Euler's Method, Improved Euler's Method, and a 4th order Runge-Kutta method to approximate $y(2)$. Compare your results with the actual value of $y(2)$. Which method produces the best results?

- (d) Repeat part (c) using the initial condition $y(3) = 3$ and approximate $y(4)$.

2. (2 points) Use the second order Taylor formula with $h = 0.1$ to approximate $y(0.5)$ for the initial value problem

$$y' = (x + y - 1)^2, \quad y(0) = 2.$$

3. (2 points) A 50-gallon tank is filled with 10 gallons of pure water. A spigot is opened above the tank, and a salt water solution containing 0.1 lb of salt per gallon begins flowing into the tank at a rate of 5 gal/min. Simultaneously, a drain is opened at the bottom of the tank allowing the solution to leave the tank at a rate of 1 gal/min. What will be the concentration of salt in the solution at the precise moment when tank reaches its maximum capacity? Set up the differential equation for the amount of salt in the tank at time t . Use a 4th order Runge Kutta method to approximate the solution.