Math 216 - Test 3a

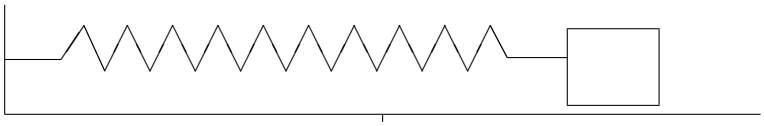
November 22, 2010

Name _____

Score _____

Show all work. Supply explanations when necessary.

1. (12 points) A 9-kg mass is attached to a spring with spring constant 2 N/m. The mass is moved 1 m to the left of equilibrium (compressing the spring) and released from rest. Assume that there are no damping forces acting on the mass-spring system. Set up and solve the initial value problem that describes the equation of motion of the mass. What is the period of the oscillations?



$x = 0$
(Equilibrium)

2. (8 points) Find the general solution of the following equation.

$$y'' - 4y' + 7y = 0$$

3. (6 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

   (a) $x(t) = 2e^{-2t} + 5te^{-2t}$

   (b) $x'' + 8x' + 17x = 0$

   (c) $x(t) = \sqrt{6}\sin(4t + \pi)$

   (d) $2x'' + 5x' + 3x = 0$

4. (8 points) Consider the following equation:

$$y'' - 12y' + 36y = (3x + 5)e^{6x}.$$

Solve the corresponding homogeneous equation. Then use your undetermined coefficients table to find the appropriate form of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.

5. (16 points) Find the general solution of the following equation.

$$2y'' + 6y' - 20y = 260 \cos x$$

Math 216 - Test 3b

Name _____

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Show all work. Supply explanations when necessary.
YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (11 points) One solution of the equation

$$xy'' - (2x+1)y' + (x+1)y = 0$$

is $y_1(x) = e^x$. Find the general solution.

2. (15 points) For $x > 0$, consider the following nonhomogeneous Cauchy-Euler equation:

$$x^2y'' + 5xy' + 4y = 5x^3.$$

(a) Solve the corresponding homogeneous equation.

(b) Use variation of parameters to solve the nonhomogeneous equation. Evaluate all integrals by hand.

3. (13 points) A 1/4-kg mass is attached to a spring with spring constant 8 N/m. The damping constant for the system is 1/4 N-sec/m. The mass is moved 1 m to the right of equilibrium (stretching the spring) and released from rest. Find the equation of motion. Write your final result in terms of a single trig function with phase shift. Graph your solution[1] and attach a copy.

$x = 0$
(Equilibrium)

[1]If you don't have a good plotting program, try one that is available online such as http://fooplot.com.

4. (11 points) Find the general solution of the following equation.

$$y^{(5)} + y''' - 2y' = 0$$