Math 216 - Quiz 1
August 26, 2015

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve the initial value problem: $y' = xe^{y-x^2}$, $y(0) = 0$

$$\frac{dy}{dx} = xe^{y}e^{-x^2}$$

$$e^{-y}\,dy = xe^{-x^2}\,dx$$

$$\int e^{-y}dy = \int xe^{-x^2}dx = -\frac{1}{2}\int e^{u}du$$

$u = -x^2$

$du = -2x\,dx$

$$-e^{-y} = -\frac{1}{2}e^{u} + C$$

$$e^{-y} = \frac{1}{2}e^{-x^2} + C$$

$$-y = \ln\left(\frac{1}{2}e^{-x^2} + c\right)$$

$$y = -\ln\left(\frac{1}{2}e^{-x^2} + c\right)$$

$$y(0) = 0 \Rightarrow C = \frac{1}{2}$$

$$y = -\ln\left(\frac{1}{2}e^{-x^2} + \frac{1}{2}\right)$$

2. (3 points) Consider the following initial value problem:

$$y' = 2xy,\ y(1) = 1.$$

Solve the IVP and find the exact value of $y(1.3)$. Then use Euler's method with $h = 0.1$ to approximate $y(1.3)$.

$$\frac{dy}{y} = 2x\,dx$$

$$\ln|y| = x^2 + C$$

$$|y| = e^{x^2+C} = Ce^{x^2}$$

$$y = Ce^{x^2}$$

$$y(1) = 1 \Rightarrow C = \frac{1}{e}$$

$$y(x) = e^{x^2-1}$$

$$y(1.3) = e^{0.69} \approx 1.9937$$

EULER'S METHOD...

$y_0 = 1$

$x_0 = 1$

$y_1 = 1 + 0.1(2)(1)(1) = 1.2$

$x_1 = 1.1$

$y_2 = 1.2 + 0.1(2)(1.1)(1.2) = 1.464$

$x_2 = 1.2$

$y_3 = 1.464 + 0.1(2)(1.2)(1.464)$

$x_3 = 1.3$

$= 1.81536$

$$y(1.3) \approx 1.81536$$

3. (2 points) Consider the ODE

$$(1 - xy)y' = 1.$$

Discuss the existence/uniqueness of solutions for various initial conditions. Then use a direction field generator (see the link on the class website) to construct the direction field.

$$\frac{dy}{dx} = f(x,y) = \frac{1}{1-xy}$$

$$\left.\begin{aligned} f(x,y) &= \frac{1}{1-xy} \\ f_y(x,y) &= \frac{x}{(1-xy)^2} \end{aligned}\right\}$$

THESE ARE CONTINUOUS EVERYWHERE EXCEPT WHERE $xy = 1$

A UNIQUE SOL'N WILL EXIST FOR ANY INITIAL CONDITION WHERE $x_0 y_0 \neq 1$.

DIRECTION FIELD IS ATTACHED. NOTICE THE VERTICAL LINE SEGMENTS AT PTS WHERE $xy = 1$.

4. (2 points) Consider the ODE

$$\frac{dy}{dx} = x^2 + y^2.$$

Discuss the existence/uniqueness of solutions for various initial conditions. Then use a direction field generator (see the link on the class website) to construct the direction field.

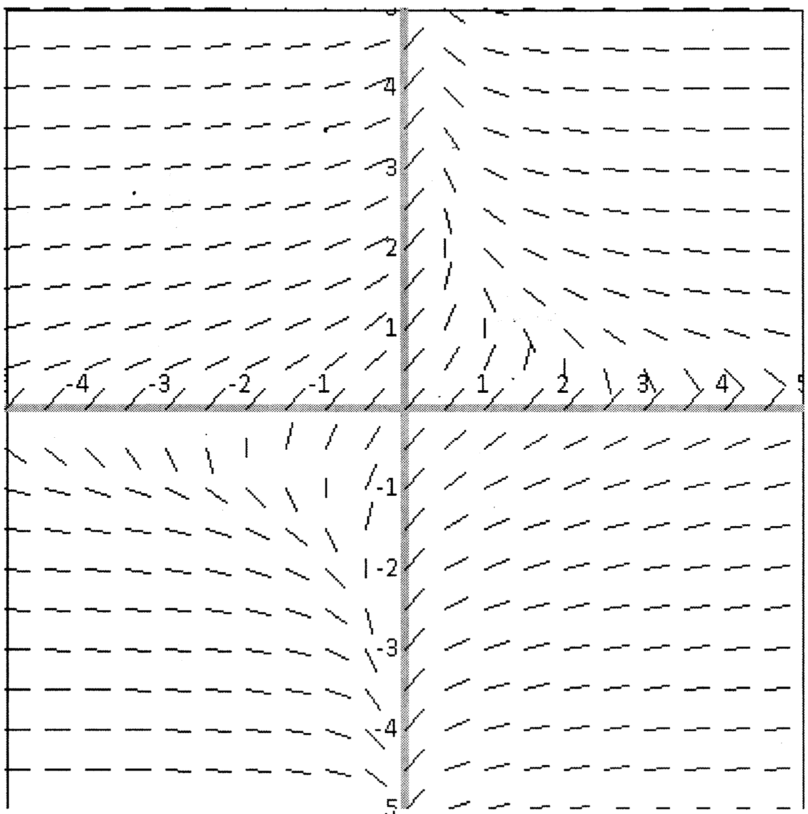
$$\frac{dy}{dx} = f(x,y) = x^2 + y^2$$

$$\left.\begin{aligned} f(x,y) &= x^2 + y^2 \\ f_y(x,y) &= 2y \end{aligned}\right\}$$

CONTINUOUS EVERYWHERE.

DIRECTION FIELD IS ATTACHED.

UNIQUE SOLUTIONS EXIST FOR ANY INITIAL CONDITION.



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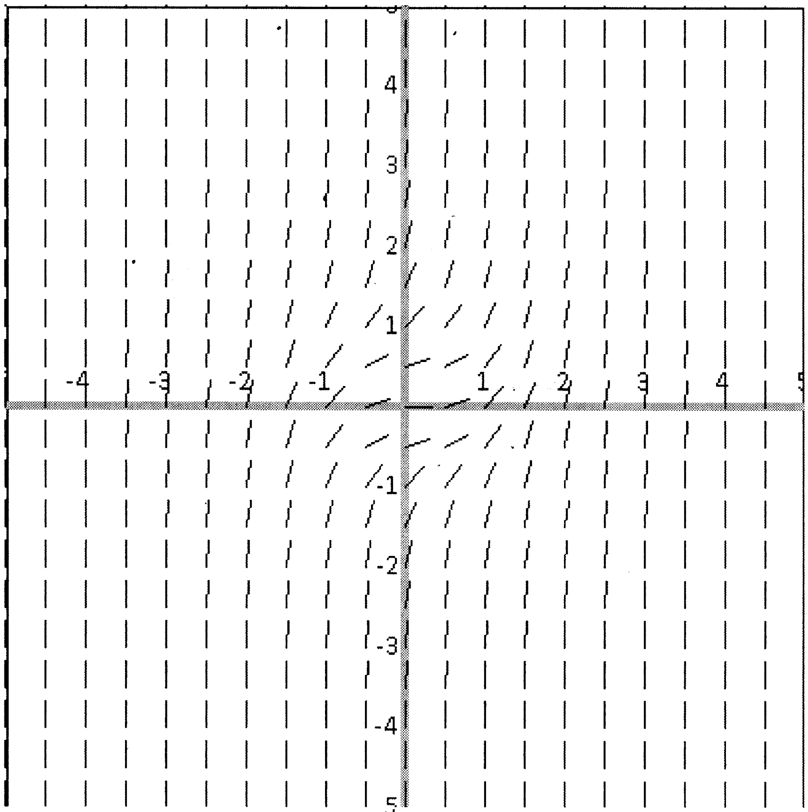
Equation : 1/(1-x*y)



Equation : x^2+y^2