Math 216 - Quiz 3

September 9, 2015

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve: $\underbrace{(xe^{xy} + e^x + 2xy + 1)}_{N(x,y)} \dfrac{dy}{dx} + \underbrace{(ye^{xy} + ye^x + y^2)}_{M(x,y)} = 0$

$$\frac{\partial N}{\partial x} = e^{xy} + xye^{xy} + e^x + 2y = \frac{\partial M}{\partial y} \Rightarrow \text{Eqn is exact!}$$

$$\frac{\partial f}{\partial y} = xe^{xy} + e^x + 2xy + 1 \Rightarrow f(x,y) = e^{xy} + ye^x + xy^2 + y + g(y)$$

$$\frac{\partial f}{\partial x} = ye^{xy} + ye^x + y^2 \Rightarrow f(x,y) = e^{xy} + ye^x + xy^2 + h(x)$$

Soln is

$$e^{xy} + ye^x + xy^2 + y = C.$$

2. (4 points) Consider the equation $(2y^2 - 6xy)\,dx + (3xy - 4x^2)\,dy = 0$. Find numbers n and m so that after each side is multiplied by $x^n y^m$, the equation is exact. Then solve.

$$(2x^N y^{m+2} - 6x^{N+1}y^{m+1})\,dx + (3x^{N+1}y^{m+1} - 4x^{N+2}y^m)\,dy = 0$$

To be exact,

$$2(m+2)x^N y^{m+1} - 6(m+1)x^{N+1}y^m = 3(N+1)x^N y^{m+1} - 4(N+2)x^{N+1}y^m$$

$$\Rightarrow \begin{matrix} 2m+4 = 3N+3 \\ -6m-6 = -4N-8 \end{matrix} \Rightarrow \begin{matrix} -3N+2m = -1 \\ 4N-6m = -2 \end{matrix} \Rightarrow \begin{matrix} N=1 \\ m=1 \end{matrix}$$

$$\frac{\partial f}{\partial x} = 2xy^3 - 6x^2y^2 \Rightarrow f(x,y) = x^2y^3 - 2x^3y^2 + g(y)$$

$$\frac{\partial f}{\partial y} = 3x^2y^2 - 4x^3y \Rightarrow f(x,y) = x^2y^3 - 2x^3y^2 + h(x)$$

Soln is $x^2y^3 - 2x^3y^2 = C.$

3. (3 points) Solve: $\sin x \dfrac{dy}{dx} + y\cos x = x \sin x, \quad y(\pi/2) = 2$

$$\frac{dy}{dx} + \cot x \; y = x$$

$$\mu(x) = e^{\int \cot x \, dx} = e^{\ln|\sin x|} = |\sin x| = \sin x$$

ASSUMING $0 < x < \pi$

$$y(x) = \frac{1}{\sin x} \int x \sin x \, dx = \frac{1}{\sin x}\left[-x\cos x + \int \cos x \, dx \right]$$

$u = x \qquad du = dx$

$dv = \sin x \, dx \qquad v = -\cos x$

$$y(x) = \frac{1}{\sin x}\left[-x \cos x + \sin x + C \right]$$

$$y(x) = -x\cot x + 1 + \frac{C}{\sin x}$$

$$y(\pi/2) = 2 \Rightarrow 2 = 1 + C \Rightarrow C = 1$$

$$y(x) = -x \cot x + 1 + \csc x, \quad 0 < x < \pi$$