Math 216 - Quiz 4
September 23, 2015

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve: $\dfrac{x(x+y)}{xy}\dfrac{dy}{dx} = \dfrac{y(x-y)}{xy}$

$$\left(\frac{x}{y}+1\right)\frac{dy}{dx} = \left(1-\frac{y}{x}\right)$$

Let $u = \frac{y}{x}$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\left(\frac{1}{u}+1\right)\left(u+x\frac{du}{dx}\right) = 1-u$$

$$(1+u)\left(u+x\frac{du}{dx}\right) = u-u^2$$

$$x\frac{du}{dx} = \frac{u-u^2}{1+u} - u = \frac{u-u^2-u(1+u)}{1+u}$$

$$x\frac{du}{dx} = \frac{-2u^2}{1+u} \Rightarrow \frac{1+u}{u^2}du = -\frac{2}{x}dx$$

$$-u^{-1} + \ln|u| = -2\ln|x| + C$$

$$2\ln|x| + \ln\left|\frac{y}{x}\right| - \frac{x}{y} = C$$

$$\boxed{\ln|x| + \ln|y| - \frac{x}{y} = C}$$

2. (3 points) Solve: $\dfrac{dy}{dx} = 2x\sqrt{y} - 2y$

$$\frac{dy}{dx} + 2y = 2x\sqrt{y}$$ divide by $\sqrt{y}$, $y \neq 0$

$$y^{-1/2}\frac{dy}{dx} + 2y^{1/2} = 2x$$

$$u = y^{1/2} \quad \frac{du}{dx} = \frac{1}{2}y^{-1/2}\frac{dy}{dx}$$

$$2\frac{du}{dx} + 2u = 2x$$

$$\frac{du}{dx} + u = x$$

$$\mu(x) = e^{\int 1\,dx} = e^x$$

$$u(x) = e^{-x}\int x e^x\,dx$$

$$= e^{-x}\left(xe^x - e^x + C\right)$$

$$= x - 1 + Ce^{-x}$$

$$y^{1/2} = x - 1 + Ce^{-x}$$

$$\boxed{y(x) = \left(x - 1 + Ce^{-x}\right)^2 \text{ or } y(x) = 0}$$

3. (4 points) According to Newton's Law of Cooling, the temperature T at time t of an object cooling in a medium of constant temperature M is described by the differential equation

$$\frac{dT}{dt} = k(M - T),$$

where k is some constant.

(a) Solve the differential equation.

$$\frac{1}{M-T}\,dT = k\,dt$$

$$-\ln|M-T| = kt + C$$

$$|M-T| = e^{-kt+C}$$

$$|M-T| = Ce^{-kt}$$

$$M-T = Ce^{-kt}$$

$$T(t) = M - Ce^{-kt}$$

(b) An object at 180°F is moved into a large room with an ambient temperature of 75°F. The object cools to 100°F in 10 min. Use your result from part (a) to find a formula for the temperature of the object at time t.

$$T(t) = 75 - Ce^{-kt}$$

$$T(0) = 180 \Rightarrow 75 - C = 180$$

$$C = -105$$

$$T(t) = 75 + 105e^{-kt}$$

$$T(10) = 100 \Rightarrow$$

$$100 = 75 + 105e^{-10k}$$

$$\frac{25}{105} = e^{-10k}$$

$$\ln\frac{25}{105} = -10k$$

$$k = \frac{\ln\frac{25}{105}}{-10}$$

$$T(t) = 75 + 105e^{\frac{\ln\frac{25}{105}}{10}t}$$

$$\approx 75 + 105e^{-0.1435t}$$