Math 216 - Quiz 6

October 7, 2015

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Use improved Euler's method with $h = 0.1$ to approximate $y(0.2)$.

$$\frac{dy}{dx} = e^{xy}, \quad y(0) = 1$$

$x_0 = 0$

$y_0 = 1$

$y^* = 1 + 0.1e^0 = 1.1$

$y_1 = 1 + \frac{0.1}{2}\left(e^0 + e^{(0.1)(1.1)}\right)$

$= 1.1058139 04 \ldots$

$x_1 = 0.1$

$y^* = 1.10\ldots + 0.1e^{0.1(1.10\ldots)}$

$= 1.2175\ldots$

$y_2 = 1.10\ldots + \frac{0.1}{2}\left(e^{(0.1)(1.10\ldots)} + e^{(0.2)(1.21\ldots)}\right)$

$= 1.2254456 67\ldots$

$y(0.2) \approx 1.23$

2. (5 points) Derive the Taylor method of order 3 for the ODE

$$\frac{dy}{dx} = xe^y.$$

Then use $h = 0.1$ and $y(0) = 2$ to approximate $y(0.2)$.

$f(x,y) = xe^y$

$f'(x,y) = e^y + xe^y\frac{dy}{dx}$

$= e^y + (xe^y)^2$

$f''(x,y) = e^y\frac{dy}{dx} + 2(xe^y)(e^y + (xe^y)^2)$

$= xe^{2y} + 2xe^{2y} + 2x^3e^{3y}$

$y_{N+1} = y_N + h\,x_N e^{y_N} + \frac{h^2}{2}\left(e^{y_N} + x_N^2 e^{2y_N}\right) + \frac{h^3}{6}\left(3x_N e^{2y_N} + 2x_N^3 e^{3y_N}\right)$

$x_{N+1} = x_N + h$

$y_0 = 2$

$x_0 = 0$

$y_1 = 2 + 0.1(0) + \frac{0.1^2}{2}(7.389\ldots) + \frac{0.1^3}{6}(0)$

$= 2.03694528\ldots$

$x_1 = 0.1$

$y_2 = 2.0369\ldots + 0.1(0.7667\ldots) + \frac{0.1^2}{2}(8.255\ldots) + \frac{0.1^3}{6}(18.536\ldots)$

$= 2.157981327\ldots$

$y(0.2) \approx 2.16$

3. (3 points) The solution of the initial value problem

$$\frac{dy}{dx} = y^2 - 2e^x y + e^{2x} + e^x, \quad y(0) = 3$$

has a vertical asymptote at a point in the interval $[0, 2]$. By experimenting with a 4th or 5th-order Runge-Kutta method, approximate this point.

USING RK5 TO OBSERVE $y \to \infty$ OR $y \to -\infty$

$h = 0.1 \to$ LOOKS LIKE ABOUT $X = 0.6$

$h = 0.01 \to$ LOOKS LIKE ABOUT $X = 0.5$

$h = 0.001 \to$ LOOK LIKE ABOUT $X = 0.5$

THE VERTICAL ASYMPTOTE

IS NEAR $X = 0.5$.