

# Math 216 - Quiz 7

October 28, 2015

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (6 points) Solve the initial value problem:  $y'' + y = 4x + 10 \sin x$ ;  $y(\pi) = 0, y'(\pi) = 2$

Homo eqn:  $y'' + y = 0$

$$r^2 + 1 = 0$$

$$r = \pm i \Rightarrow \alpha = 0, \beta = 1$$

$$y_h(x) = c_1 \cos x + c_2 \sin x$$

Nonhomo eqn #1:  $g(x) = 4x$

$$\Rightarrow y_{p_1}(x) = Ax + B$$

$$y'_{p_1}(x) = A$$

$$y''_{p_1}(x) = 0 \Rightarrow Ax + B = 4x$$

$$A = 4$$

$$B = 0$$

$$y_{p_1}(x) = 4x$$

Nonhomo eqn #2:  $g(x) = 10 \sin x$

$$\Rightarrow y_{p_2}(x) = x(A \cos x + B \sin x)$$

$$y'_{p_2}(x) = A \cos x - Ax \sin x + B \sin x + Bx \cos x$$

$$y''_{p_2}(x) = -A \sin x - A \sin x - Ax \cos x + B \cos x + B \cos x - Bx \sin x$$

$$y''_{p_2}(x) + y_{p_2}(x) = 10 \sin x$$

$$\Rightarrow -2A = 10 \quad A = -5$$

$$2B = 0 \quad B = 0$$

$$y_{p_2}(x) = -5x \cos x$$

$$y(x) = c_1 \cos x + c_2 \sin x + 4x - 5x \cos x$$

$$y(\pi) = 0 \Rightarrow -c_1 + 4\pi + 5\pi = 0$$

$$c_1 = 9\pi$$

$$y'(x) = -c_1 \sin x + c_2 \cos x + 4 - 5 \cos x + 5x \sin x$$

$$y'(\pi) = 2 \Rightarrow -c_2 + 4 + 5 = 2$$

$$c_2 = 7$$

$$y(x) = 9\pi \cos x + 7 \sin x + 4x - 5x \cos x$$

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2. (4 points) Solve the following nonhomogeneous, Cauchy-Euler equation.

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \ln x$$

$x = e^t$  gives

$$y'' - 2y' + y = t$$

Homo eqn:

$$y'' - 2y' + y = 0$$

$$(r-1)^2 = 0 \quad r=1, r=1$$

$$y_h(t) = c_1 e^t + c_2 t e^t$$

Non homo eqn:

$$g(t) = t$$

$$y_p(t) = At + B$$

$$y_p'(t) = A$$

$$y_p''(t) = 0$$

$$0 - 2A + At + B = t$$

$$A = 1$$

$$B = 2$$

$$y_p(t) = t + 2$$

$$y(t) = c_1 e^t + c_2 t e^t + t + 2$$

$$y(x) = c_1 x + c_2 x \ln x + 2 + \ln x$$