

# Math 216 - Quiz 8

November 4, 2015

Name key

Score \_\_\_\_\_

Show all work. Supply explanations when necessary.

1. (5 points) Solve:  $4y'' + 36y = x + \csc 3x$

Homo eqn:

$$y'' + 9y = 0$$

$$r^2 + 9 = 0$$

$$r = \pm 3i; \alpha = 0, \beta = 3$$

$$y_h(x) = c_1 \cos 3x + c_2 \sin 3x$$

Nonhomo eqn #1:  $4y'' + 36y = x$

$$g(x) = x$$

$$y_p(x) = Ax + B$$

$$y_p'(x) = A, y_p''(x) = 0$$

$$36Ax + 36B = x$$

$$\Rightarrow A = \frac{1}{36}, B = 0$$

$$y_p(x) = \frac{1}{36}x$$

Nonhomo eqn #2:  $y'' + 9y = \frac{1}{4} \csc 3x$

$$g(x) = \frac{1}{4} \csc 3x$$

$$y_1(x) = \cos 3x$$

$$y_{p_2}(x) = v_1 y_1 + v_2 y_2$$

$$y_2(x) = \sin 3x$$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3$$

$$\begin{aligned} v_1(x) &= \int \frac{-\csc 3x \sin 3x}{12} dx \\ &= \int -\frac{1}{12} dx = -\frac{1}{12}x \end{aligned}$$

$$\begin{aligned} v_2(x) &= \int \frac{\csc 3x \cos 3x}{12} dx \\ &= \int \frac{1}{12} \cot 3x dx \\ &= \frac{1}{36} \ln |\sin 3x| \end{aligned}$$

$$y_{p_2}(x) = -\frac{1}{12}x \cos 3x + \frac{1}{36} \ln |\sin 3x| \sin 3x$$

$$\begin{aligned} y(x) &= c_1 \cos 3x + c_2 \sin 3x + \frac{1}{36}x \\ &\quad - \frac{1}{12}x \cos 3x + \frac{1}{36} \ln |\sin 3x| \sin 3x \end{aligned}$$

2. (5 points) Solve:  $y'' - 2y' + y = \frac{e^x}{1+x^2}$

Homogeneous eqn:  $y'' - 2y' + y = 0$

$$(r-1)^2 = 0 \Rightarrow r=1, r=1$$

$$y_h(x) = c_1 e^x + c_2 x e^x.$$

Nonhomogeneous eqn:  $g(x) = \frac{e^x}{1+x^2}$

$$W = \begin{vmatrix} e^x & x e^x \\ e^x & e^x + x e^x \end{vmatrix} = e^{2x}$$

$$y_p(x) = v_1 y_1 + v_2 y_2$$

$$v_1(x) = \int \frac{-\frac{e^x}{1+x^2} (x e^x)}{e^{2x}} dx$$

$$= \int \frac{-x}{1+x^2} dx$$

$$= -\frac{1}{2} \ln(1+x^2)$$

$$v_2(x) = \int \frac{\frac{e^x}{1+x^2} (e^x)}{e^{2x}} = \int \frac{1}{1+x^2} dx$$

$$= \tan^{-1} x$$

$$y_p(x) = -\frac{1}{2} e^x \ln(1+x^2) + x e^x \tan^{-1} x$$

$$y(x) = c_1 e^x + c_2 x e^x - \frac{1}{2} e^x \ln(1+x^2) + x e^x \tan^{-1} x$$