Math 216 - Quiz 9

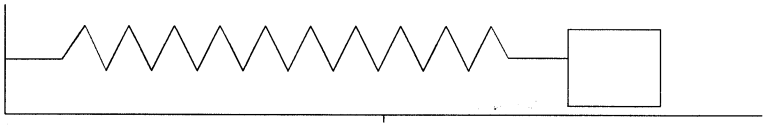
November 11, 2015

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) A 1/2-kg mass is attached to a spring with spring constant 4 N/m. The damping constant for the system is 1/2 N-sec/m. The mass is moved 1 m to the left of equilibrium (compressing the spring) and pushed to the left at 1 m/sec. Find the equation of motion. Write your final result in terms of a single trig function with phase shift. Graph your solution[1] and attach a copy.



$x = 0$
(Equilibrium)

$\frac{1}{2}x'' + \frac{1}{2}x' + 4x = 0;$

$x(0) = -1$

$x'(0) = -1$

$x'' + x' + 8x = 0$

$r^2 + r + 8 = 0$

$r = \dfrac{-1 \pm \sqrt{1 - 4(1)(8)}}{2}$

$\alpha = -\frac{1}{2}, \quad \beta = \frac{\sqrt{31}}{2}$

$x(t) = c_1 e^{-t/2} \cos\frac{\sqrt{31}}{2}t + c_2 e^{-t/2} \sin\frac{\sqrt{31}}{2}t$

$x(0) = -1 \Rightarrow c_1 = -1$

$x'(0) = -\frac{c_1}{2} + \frac{\sqrt{31}}{2}c_2$

$x'(0) = -1 \Rightarrow -1 = \frac{1}{2} + \frac{\sqrt{31}}{2}c_2$

$c_2 = \dfrac{-3}{\sqrt{31}}$

$x(t) = -e^{-t/2} \cos\frac{\sqrt{31}}{2}t + \frac{3}{\sqrt{31}} e^{-t/2} \sin\frac{\sqrt{31}}{2}t$

$A = \sqrt{(-1)^2 + \left(-\frac{3}{\sqrt{31}}\right)^2} = \sqrt{\frac{40}{31}}$

c_1 & c_2 BOTH NEG $\Rightarrow$ φ IN QUAD III

$\varphi = \tan^{-1}\left(\frac{\sqrt{31}}{3}\right) + \pi$

≈ 4.21817

$x(t) = \sqrt{\frac{40}{31}}\, e^{-t/2} \sin\left(\frac{\sqrt{31}}{2}t + 4.21817\right)$

[1] If you don't have a good plotting program, try one that is available online such as http://fooplot.com.

2*sqrt(10)*%e^-(t/2)*sin(sqrt(31)*t/2+4.218172935921613)/sqrt(31)