Math 216 - Final Exam

December 9, 2015

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Solve the following initial value problem:

$$\frac{dy}{dx} = 3xy, \quad y(0) = 5$$

2. (10 points) Find the orthogonal trajectories for the one-parameter family of curves $y = 3x^2 + C$.

3. (6 points) Consider the initial value problem $y' = xy,\ y(1) = 2$. Use Euler's method with $h = 0.5$ to approximate $y(2)$.

4. (8 points) Determine the recursive formulas for the Taylor method of order 3 for the IVP

$$\frac{dy}{dx} = x + 2y, \quad y(0) = 1.$$

5. (12 points) Solve the initial value problem

$$y'' + 2y' + 4y = 0; \quad y(0) = 2,\ y'(0) = -1$$

6. (12 points) Find the general solution of the ODE

$$(x^2 + 1)y' + xy = 2x.$$

7. (12 points) Find an integrating factor for the differential equation

$$(x + xy^3)\,dx + 3y^2\,dy = 0.$$

Then use your integrating factor to solve the equation.

8. (16 points) Solve the following initial value problem:

$$y'' - 5y' + 4y = 2e^{4x}; \quad y(0) = 1,\ y'(0) = 2$$

9. (16 points) Consider the initial value problem

$$x'' + 9x = e^t; \quad x(0) = 2,\ x'(0) = 1.$$

(a) Compute the Laplace transform of both sides of the equation. Then solve for $X(s)$, the Laplace transform of $x(t)$.

(b) After expanding your solution above, you would find that

$$X(s) = \frac{1}{10}\left(\frac{1}{s-1}\right) + \frac{19}{10}\left(\frac{s}{s^2+9}\right) + \frac{9}{10}\left(\frac{1}{s^2+9}\right).$$

Compute the inverse transform of $X(s)$ to determine $x(t)$.

10. (16 points) Find the general solution of $y'' + 4y = \sec 2x$.

11. (12 points) An object is launched from the ground into the air so that its velocity, in feet per second, at any time t (in seconds) satisfies the initial value problem

$$v' = -27v - 32, \quad v(0) = 20.$$

Determine the function that gives the height of the object at time t.

12. (20 points) Consider the following system of ODEs.

$$x' + y' - x = 5$$

$$x' + y' + y = 1$$

(a) Write the system in operator notation and then determine the number of arbitrary constants in the general solution.

(b) Use any method to solve the system.