Solving 1st order linear ODEs

- Given $y'(x) + p(x)\, y(x) = q(x)$

- Find the integrating factor

$$\mu(x) = e^{\int p(x)\, dx}$$

- The solution follows from

$$\mu(x)\, y(x) = \int \mu(x)\, q(x)\, dx$$

(Don't forget your constant of integration!)

- Or, if you are given an initial condition...

$$\mu(x)\, y(x) = \mu(x_0)\, y(x_0) + \int_{x_0}^{x} \mu(t)\, q(t)\, dt$$

Suppose $p(x)$ and $q(x)$ are continuous on an interval (a, b) containing x_0. Then for any choice of initial value y_0, the linear IVP

$$\frac{dy}{dx} + p(x)y = q(x), \quad y(x_0) = y_0$$

has a unique solution on the entire interval (a, b).

A first order equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n,$$

where P and Q are continous on an interval and n is a real number, is called a *Bernoulli equation*.

Divide both sides of the equation by y^n and then make the substitution $u = y^{1-n}$. This will transform the Bernoulli equation into the linear equation

$$\left(\frac{1}{1-n}\right)\frac{du}{dx} + P(x)u = Q(x).$$

Also note that $y = 0$ solves the original Bernoulli equation.