The differential form $M(x, y)dx + N(x, y)dy$ is said to be *exact* on a rectangle R if it is the total differential of a function $F(x, y)$ on R. That is, $M(x, y)dx + N(x, y)dy$ is exact if

$$M(x, y) = \frac{\partial F}{\partial x} \qquad N(x, y) = \frac{\partial F}{\partial y}$$

for some function $F(x, y)$ on R.

If $M(x, y)dx + N(x, y)dy$ is an exact differential form, then the differential equation

$$M(x, y)dx + N(x, y)dy = 0$$

is called an *exact equation*.

Test for exactness

Suppose the first partial derivatives of $M(x, y)$ and $N(x, y)$ are continuous in a rectangle R. Then

$$M(x, y)dx + N(x, y)dy = 0$$

is an exact differential equation if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

for all (x, y) in R.