Finding an Antiderivative through a Given Point

There are two popular approaches to finding the antiderivative of a given function that satisifes a given condition. Consider the problem of finding $f(x)$ if

$$f'(x) = 6x^2 + 2x - 5 \quad \text{and} \quad f(1) = 3.$$

Approach #1 Use an indefinite integral and then solve for the constant of integration:

$$f(x) = \int (6x^2 + 2x - 5)\, dx = 2x^3 + x^2 - 5x + C$$

$$f(1) = 3 \Longrightarrow 2(1)^3 + (1)^2 - 5(1) + C = 3$$

$$-2 + C = 3 \Longrightarrow C = 5$$

$$f(x) = 2x^3 + x^2 - 5x + 5$$

Approach #2 Use a definite integral:

$$f(x) = f(1) + \int_1^x f'(t)\, dt$$

$$f(x) = f(1) + \int_1^x (6t^2 + 2t - 5)\, dt$$

$$f(x) = 3 + (2t^3 + t^2 - 5t)\Big|_1^x$$

$$f(x) = 3 + 2x^3 + x^2 - 5x - (-2)$$

$$f(x) = 2x^3 + x^2 - 5x + 5$$

While it may seem more complicated, the second approach is especially nice because it gives a formula for $f(x)$ in only one step.

You should be comfortable with both of these approaches.