# Math 216 - Quiz 2

January 25, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. Consider the following initial value problem:

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1$$

(a) (1 point) Find the slope of the solution curve passing through $(0, -1)$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(0,-1)} = \frac{(0)(-1)^3}{\sqrt{1+0^2}} = \boxed{0}$$

(b) (3 points) Use Euler's method with $h = 0.1$ to approximate $y(0.3)$. Show all steps.

$y_0 = -1, \quad x_0 = 0$

$y_1 = y_0 + h f(x_0, y_0) = -1 + 0.1(0) = -1$ ← From above

$x_1 = 0.1$

$y(0.1) \approx -1$

$y_2 = y_1 + h f(x_1, y_1) = -1 + 0.1\left(\frac{0.1(-1)^3}{\sqrt{1+0.1^2}}\right) = -1.00995037\ldots$

$x_2 = 0.2$

$y(0.2) \approx -1.00995$

$y_3 = y_2 + h f(x_2, y_2) = -1.00995 + 0.1\left(\frac{0.2(-1.00995)^3}{\sqrt{1+0.2^2}}\right) = -1.030153\ldots$

$x_3 = 0.3$

$\boxed{y(0.3) \approx -1.03015}$

(c) (3 points) Find the exact solution of the initial value problem.

$$\frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}}\,dx$$

$$\int y^{-3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$$-\frac{1}{2}y^{-2} = \frac{1}{2}\int u^{-1/2}\,du$$

$$-\frac{1}{y^2} = 2u^{1/2} + C$$

$$\frac{1}{y^2} = C - 2\sqrt{1+x^2}$$

$y(0) = -1 \Rightarrow 1 = C - 2 \Rightarrow C = 3$

$$\frac{1}{y^2} = 3 - 2\sqrt{1+x^2}$$

$$y(x) = \frac{-1}{\sqrt{3-2\sqrt{1+x^2}}}$$

↑ WE NEED THE NEGATIVE $\sqrt{\ }$ TO SATISFY THE INITIAL CONDITION.

(d) (2 points) Use your solution to find the exact value of $y(0.3)$. Then find the percent error in your approximation from Euler's method.

$$y(0.3) = \frac{-1}{\sqrt{3-2\sqrt{1.09}}} = -1.047169965\ldots$$

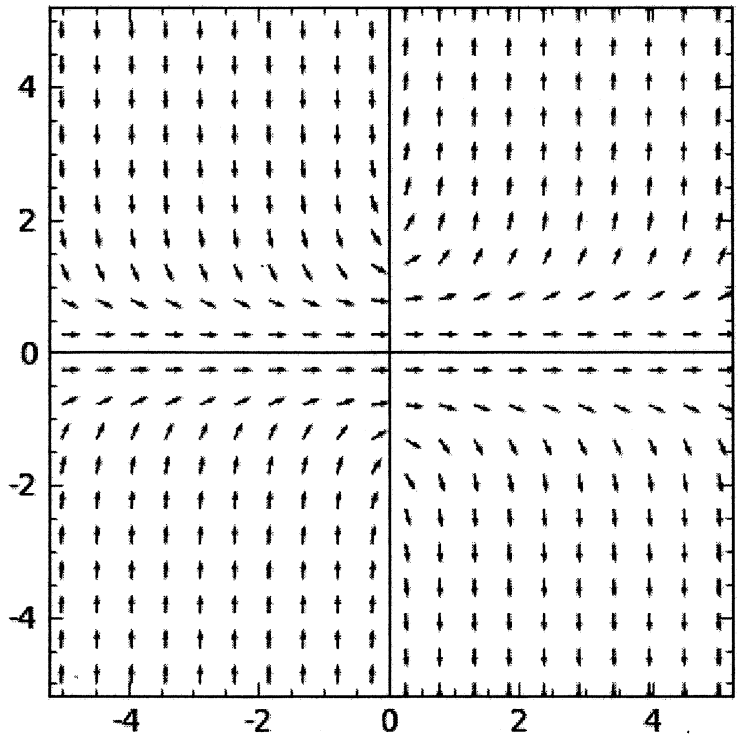
$$\approx \boxed{-1.04717}$$

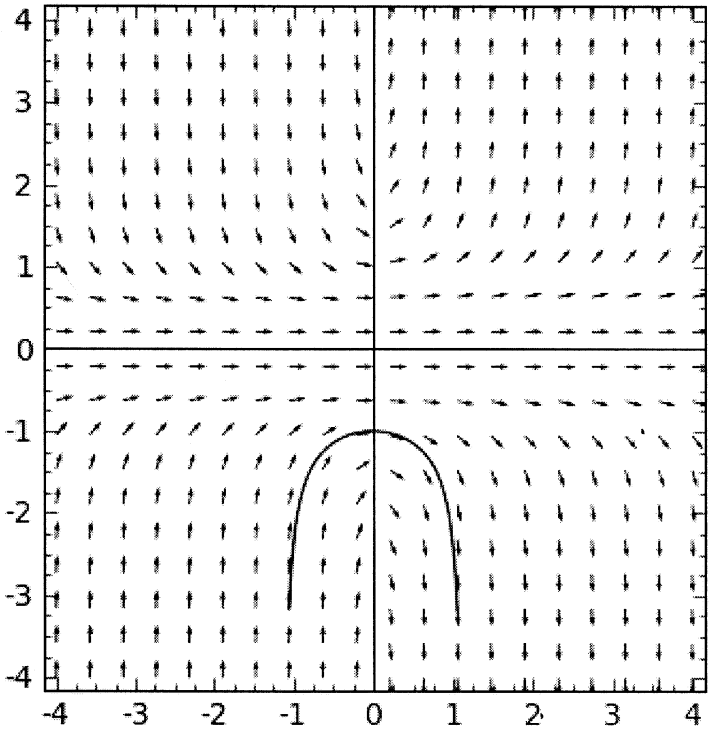
$$\%\ \text{error} = \frac{|-1.04717 - (-1.03015)|}{|-1.04717|} = 0.01625\ldots$$

$$\approx \underline{1.6\%}$$

(e) (1 point) Use the direction field shown below to draw a rough sketch of the solution curve passing through $(0, -1)$.

See next page

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