Math 216 - Quiz 3

February 1, 2012

Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve: $\dfrac{dx}{dt} = \dfrac{t}{xe^{t+2x}}, \quad x(0) = 2$

$$\frac{dx}{dt} = \frac{te^{-t}}{xe^{2x}}$$

$$xe^{2x}\,dx = te^{-t}\,dt$$

$$\int xe^{2x}\,dx = \int te^{-t}\,dt$$

+	x	e^{2x}
−	1	$\frac{1}{2}e^{2x}$
+	0	$\frac{1}{4}e^{2x}$

+	t	e^{-t}
−	1	$-e^{-t}$
+	0	e^{-t}

$$\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} = -te^{-t} - e^{-t} + C$$

$$x(0) = 2 \Rightarrow e^4 - \frac{1}{4}e^4 = -1 + C$$

$$\frac{3}{4}e^4 + 1 = C$$

$$\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} = -te^{-t} - e^{-t} + 1 + \frac{3}{4}e^4$$

MULT BOTH SIDES BY 4...

$$\boxed{e^{2x}(2x-1) = -4e^{-t}(t+1) + 4 + 3e^4}$$

2. (3 points) Section 2.2, Page 45, Problem #36

$$T(t) = M + Ce^{kt}$$

① $T(0) = 100 = M + C$

② $T(5) = 80 = M + Ce^{5k}$

③ $T(10) = 65 = M + Ce^{10k}$

SOLVE FOR M, C, k

LET $u = e^{5k}$

① − ② : $C - Cu = 20$

① − ③ : $\underbrace{C - Cu^2}= 35$

$$(C - Cu)(1+u) = 35$$

$$20(1+u) = 35$$

$$1 + u = \frac{35}{20}$$

$$u = \frac{3}{4}$$

$$e^{5k} = \frac{3}{4}$$

$$k = \frac{\ln 3/4}{5}$$

$$C(1-u) = 20$$

$$\Rightarrow C = 80$$

$$100 = M + C$$

$$\Rightarrow \boxed{M = 20^\circ}$$

$p(x) = \frac{3}{x}, \quad q(x) = 3x - 2$

3. (3 points) Solve: $\dfrac{dy}{dx} + \dfrac{3y}{x} + 2 = 3x, \quad y(1) = 1$

$$\int \frac{3}{x}\,dx = 3\ln|x|$$

$$\mu(x) = e^{3\ln|x|} = |x|^3$$

BASED ON THE INITIAL CONDITION AND $X \neq 0$, LET'S ASSUME $X > 0$.

$$\mu(x) = x^3$$

$$x^3 y(x) = \int x^3(3x-2)\,dx$$

$$x^3 y(x) = \frac{3}{5}x^5 - \frac{2}{4}x^4 + C$$

$$y(1) = 1 \Rightarrow 1 = \frac{3}{5} - \frac{2}{4} + C$$

$$C = \frac{9}{10}$$

$$y(x) = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{9}{10x^3}$$

4. (1 point) Section 1.2, Page 14, Problem #28

$$\frac{dy}{dx} = \underbrace{3x - \sqrt[3]{y-1}}, \quad y(2) = 1$$

$$f(x,y) = 3x - \sqrt[3]{y-1}$$

f IS CONTINUOUS EVERYWHERE.

$$\frac{\partial f}{\partial y} = -\frac{1}{3}(y-1)^{-2/3} = \frac{-1}{3\sqrt[3]{(y-1)^2}}$$

$\frac{\partial f}{\partial y}$ IS <u>NOT</u> DEFINED AT $y = 1$

THM DOES NOT APPLY!