# Math 216 - Quiz 4

February 7, 2012

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2.5 points) Solve, assuming $x > 1$: $\quad x \ln x\, y' + y = 3x^3, \quad y(e) = 0$

$$\frac{dy}{dx} + \frac{1}{x \ln x} y = \frac{3x^3}{\ln x}$$

$$\mu(x) = e^{\int \frac{1}{x \ln x} dx} = e^{\ln|\ln x|} = \ln x$$

$$(\ln x) y = \int 3x^2 dx$$

$$(\ln x) y = x^3 + C$$

$$y = \frac{x^3}{\ln x} + \frac{C}{\ln x}$$

$y(e) = 0$

$\Rightarrow \quad e^3 + C = 0$

$\Rightarrow \quad C = -e^3$

$$y(x) = \frac{x^3 - e^3}{\ln x}$$

2. (2.5 points) Solve: $\quad e^y\, dx + (xe^y + 2y)\, dy = 0$

$$\frac{\partial M}{\partial y} = e^y = \frac{\partial N}{\partial x} = e^y \Rightarrow \text{Equation is exact.}$$

$$\frac{\partial F}{\partial x} = e^y \Rightarrow F(x,y) = xe^y + g(y)$$

$$\frac{\partial F}{\partial y} = xe^y + 2y \Rightarrow F(x,y) = xe^y + y^2 + h(x)$$

$$F(x,y) = xe^y + y^2$$

Solution is:

$$xe^y + y^2 = C$$

3. (2.5 points) Page 67, Problem #10

$$(2y^2 + 2y + 4x^2)\,dx + (2xy + x)\,dy = 0$$

$$\frac{\partial M}{\partial y} = 4y + 2 \neq \frac{\partial N}{\partial x} = 2y + 1$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y+1}{x(2y+1)} = \frac{1}{x}$$

$$\mu(x) = e^{\int \frac{1}{x}\,dx} = e^{\ln|x|} = |x| = x$$

ASSUMING $x > 0$

MULT BY INTEGRATING FACTOR...

$$(2xy^2 + 2xy + 4x^3)\,dx + (2x^2y + x^2)\,dy = 0$$

$$\frac{\partial F}{\partial x} = 2xy^2 + 2xy + 4x^3$$

$$\Rightarrow F(x,y) = x^2y^2 + x^2y + x^4 + g(y)$$

$$\frac{\partial F}{\partial y} = 2x^2y + x^2$$

$$\Rightarrow F(x,y) = x^2y^2 + x^2y + h(x)$$

$$F(x,y) = x^2y^2 + x^2y$$

SOLUTION IS $x^2y^2 + x^2y + x^4 = C$

4. (2.5 points) Page 67, Problem #14

$$\underbrace{x^N y^m (12 + 5xy)}_{M(x,y)}\,dx + \underbrace{x^N y^m \left(\frac{6x}{y} + 3x^2\right)}_{N(x,y)}\,dy = 0$$

$$\frac{\partial M}{\partial y} = 12m x^N y^{m-1} + 5(m+1)x^{N+1}y^m$$

$$\frac{\partial N}{\partial x} = 6(N+1)x^N y^{m-1} + 3(N+2)x^{N+1}y^m$$

THESE ARE EQUAL IF

$12m = 6(N+1)$

$5(m+1) = 3(N+2)$

SOLUTION IS $m = 2,\ N = 3$

INT FACTOR IS $x^3y^2$...

$$(12x^3y^2 + 5x^4y^3)\,dx + (6x^4y + 3x^5y^2)\,dy = 0$$

$$\partial F/\partial x = 12x^3y^2 + 5x^4y^3 \Rightarrow F(x,y) = 3x^4y^2 + x^5y^3 + g(y)$$

$$\partial F/\partial y = 6x^4y + 3x^5y^2 \Rightarrow F(x,y) = 3x^4y^2 + x^5y^3 + h(x)$$

SOLUTION IS $3x^4y^2 + x^5y^3 = C$.