Math 216 - Quiz 5

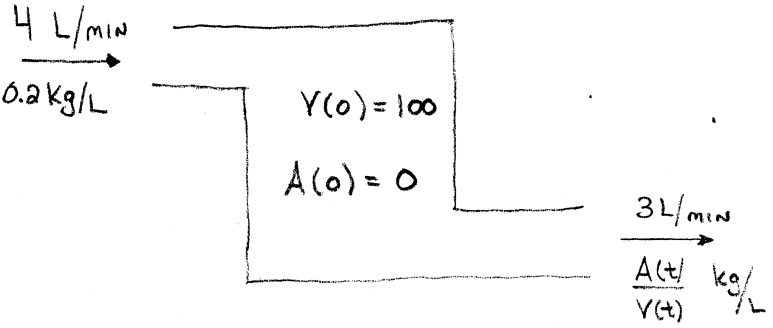
February 22, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2.5 points) Page 99, Problem #4



$A(t)$ = AMOUNT OF SALT IN kg AT TIME t

$V(t)$ = VOLUME OF TANK IN L AT TIME t

$= 100 + t$

$$\frac{dA}{dt} = 0.8 - \frac{3A}{100+t}, \quad A(0) = 0$$

$$\frac{dA}{dt} + \frac{3}{100+t}A = 0.8$$

$$\mu(t) = e^{\int \frac{3}{100+t}\,dt} = (100+t)^3$$

$$(100+t)^3 A = \int 0.8(100+t)^3\,dt$$

$$= 0.2(100+t)^4 + C$$

$$A(t) = 0.2(100+t) + \frac{C}{(100+t)^3}$$

$$A(0) = 0 = 20 + \frac{C}{100^3} \Rightarrow C = -20(100^3)$$

$$A(t) = 0.2(100+t) - \frac{20(100^3)}{(100+t)^3}$$

$$\frac{A(t)}{V(t)} = 0.2 - \frac{20(100^3)}{(100+t)^4} = 0.1$$

$$\Rightarrow \boxed{t \approx 18.92 \text{ min}}$$

2. (2.5 points) Page 101, Problem #24

EXPONENTIAL DECAY...

$$A(t) = A_0 e^{kt}, \quad k < 0$$

$$A(t) = 300e^{kt}$$

$$A(5) = 200 = 300e^{5k}$$

$$\ln \tfrac{2}{3} = 5k$$

$$k = \frac{\ln 2/3}{5}$$

$$10 = 300e^{kt}$$

$$\ln \tfrac{1}{30} = kt$$

$$t = \frac{\ln 1/30}{k} = \frac{\ln 1/30}{\frac{\ln 2/3}{5}} = \frac{5 \ln 1/30}{\ln 2/3}$$

$$\approx \boxed{41.94 \text{ yrs}}$$

3. (2.5 points) Page 115, Problem #6

$$8\frac{dv}{dt} = -8g - 16v, \quad v(0) = 20, \quad x(0) = 100$$

$$\frac{dv}{dt} = -g - 2v$$

$$\frac{1}{g+2v}\,dv = -dt$$

$$\frac{1}{2}\ln|g+2v| = -t + C$$

$$\ln|g+2v| = -2t + C$$

$$|g+2v| = Ce^{-2t}$$

$$g + 2v = Ce^{-2t}$$

$$2v = Ce^{-2t} - g$$

$$v(t) = Ce^{-2t} - \frac{1}{2}g$$

$$v(0) = 20 = C - \frac{1}{2}g \Rightarrow C = 20 + \frac{1}{2}g$$

$$v(t) = \left(20 + \frac{1}{2}g\right)e^{-2t} - \frac{1}{2}g$$

$$x(t) = -\frac{1}{2}\left(20 + \frac{1}{2}g\right)e^{-2t} - \frac{1}{2}gt + C$$

$$x(0) = 100 \Rightarrow -\frac{1}{2}\left(20 + \frac{1}{2}g\right) + C = 100$$

$$\Rightarrow C = 100 + \frac{1}{2}\left(20 + \frac{1}{2}g\right)$$

$$x(t) = \left(-10 - \frac{1}{4}g\right)e^{-2t} - \frac{1}{2}gt + 110 + \frac{1}{4}g$$

$$x(t) = 0 \Rightarrow \boxed{t = 22.93 \text{ sec}}$$

NEWTON'S METHOD.

(2.5 points) Page 107, Problem #4

$T(t) = M - Ce^{kt}$ (DERIVATION FROM CLASS)

$T(0) = 10$

$M = 23$

$T(10) = 15$

FIND t WHEN $T(t) = 18$

$$T(t) = 23 - 13e^{kt}$$

$$T(10) = 15 \Rightarrow 23 - 13e^{10k} = 15$$

$$8 = 13e^{10k}$$

$$\ln\frac{8}{13} = 10k$$

$$k = \frac{\ln 8/13}{10}$$

$$18 = 23 - 13e^{kt}$$

$$\frac{5}{13} = e^{kt}$$

$$\ln\frac{5}{13} = kt$$

$$t = \frac{\ln\frac{5}{13}}{\frac{\ln 8/13}{10}} = \frac{10\ln 5/13}{\ln 8/13}$$

$$\approx \boxed{19.68 \text{ MIN}}$$