

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Find the orthogonal trajectories for the family of curves described by the equation $x^2 + y^2 = 2Cx$.

$$\frac{x^2 + y^2}{x} = 2C \Rightarrow \frac{x(2x + 2y \frac{dy}{dx}) - (x^2 + y^2)}{x^2} = 0$$

$$\Rightarrow 2x^2 + 2xy \frac{dy}{dx} = x^2 + y^2$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \Rightarrow \text{ORTHO TRAJ'S SATISFY}$$

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

$$2xy dx + (y^2 - x^2) dy = 0$$

$$\frac{\partial M}{\partial y} = 2x, \quad \frac{\partial N}{\partial x} = -2x \quad \text{NOT EXACT,}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-N(x,y)} = \frac{4x}{-2xy} = -\frac{2}{y}$$

BUT EXACT WITH INTEGRATING FACTOR

$$\mu(y) = e^{\int -\frac{2}{y} dy} = \frac{1}{y^2}$$

$$\frac{2x}{y} dx + \left(1 - \frac{x^2}{y^2}\right) dy = 0$$

$$\frac{\partial F}{\partial x} = \frac{2x}{y} \Rightarrow F(x,y) = \frac{x^2}{y} + g(y)$$

$$\frac{\partial F}{\partial y} = \left(1 - \frac{x^2}{y^2}\right) \Rightarrow F(x,y) = y + \frac{x^2}{y} + h(x)$$

$$F(x,y) = y + \frac{x^2}{y}$$

$$y + \frac{x^2}{y} = C$$

$$y^2 + x^2 = Cy$$

2. (4 points) Determine the recursive formulas for the Taylor method of order 4 for the initial value problem

$$y' = x^2 + y, \quad y(0) = 0.$$

Then take two steps using stepsize $h = 0.1$.

$$f(x,y) = x^2 + y, \quad f'(x,y) = 2x + \frac{dy}{dx} = 2x + x^2 + y, \quad f''(x,y) = 2 + 2x + \frac{dy}{dx} \\ = 2 + 2x + x^2 + y$$

$$f'''(x,y) = 2 + 2x + \frac{dy}{dx} = 2 + 2x + x^2 + y$$

$$y_{N+1} = y_N + h(x_N^2 + y_N) + \frac{h^2}{2}(2x_N + x_N^2 + y_N) + \frac{h^3}{6}(2 + 2x_N + x_N^2 + y_N) \\ + \frac{h^4}{24}(2 + 2x_N + x_N^2 + y_N)$$

$$x_{N+1} = x_N + h$$

Two steps...

$$y_0 = 0$$

$$x_0 = 0$$

$$y_1 = 0 + 0.1(0+0) + \frac{0.01}{2}(0+0+0) + \frac{0.001}{6}(2+0+0+0) + \frac{0.0001}{24}(2+0+0+0) \\ = 0.000341\bar{6}$$

$$x_1 = 0.1$$

$$y_2 = y_1 + 0.1(0.01 + y_1) + \frac{0.01}{2}(0.2 + 0.01 + y_1) + \frac{0.001}{6}(2 + 0.2 + 0.01 + y_1) \\ + \frac{0.0001}{24}(2 + 0.2 + 0.001 + y_1) = 0.0028126\dots$$

$$x_2 = 0.2$$

$$y(0.2) \approx 0.0028$$

3. (2 points) Page 139, Problem #12

FIND THE MAX VALUE OF y ON $[1,2]$

$$y' = \frac{1.8}{x^4} - y^2, \quad y(1) = -1$$

MAX VALUE IS APPROX

$$y = -0.9282$$

$$\text{AT } x = 1.2$$