Math 216 - Quiz 7

March 7, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Page 165, Problem #8

$z'' + z' - z = 0$

CHAR. EQ. IS $r^2 + r - 1 = 0$

$$r = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$r_1 = \frac{-1+\sqrt{5}}{2}, \quad r_2 = \frac{-1-\sqrt{5}}{2}$$

$$z(x) = c_1 e^{\left(\frac{-1+\sqrt{5}}{2}\right)x} + c_2 e^{\left(\frac{-1-\sqrt{5}}{2}\right)x}$$

2. (3 points) Page 165, Problem #16

$y'' - 4y' + 3y = 0; \quad y(0) = 1, \; y'(0) = 1/3$

CHAR. EQ. IS $r^2 - 4r + 3 = 0$

$(r-3)(r-1) = 0$

$r_1 = 3, \; r_2 = 1$

$y(x) = c_1 e^{3x} + c_2 e^{x}$

$y(0) = 1 \Rightarrow c_1 + c_2 = 1$

$y'(x) = 3c_1 e^{3x} + c_2 e^{x}$

$y'(0) = \frac{1}{3} \Rightarrow 3c_1 + c_2 = \frac{1}{3}$

$c_1 + c_2 = 1$

$3c_1 + c_2 = \frac{1}{3}$

$-2c_1 = \frac{2}{3} \Rightarrow c_1 = -\frac{1}{3}$

$c_2 = \frac{4}{3}$

$$y(x) = -\frac{1}{3}e^{3x} + \frac{4}{3}e^{x}$$

3. (3 points) Page 166, Problem #44

$$y''' - 2y'' - y' + 2y = 0; \quad y(0) = 2,\ y'(0) = 3,\ y''(0) = 5$$

CHAR EQ IS $r^3 - 2r^2 - r + 2 = 0$

$$r^2(r-2) - 1(r-2) = 0$$
$$(r-2)(r^2-1) = 0$$
$$(r-2)(r-1)(r+1) = 0$$
$$r_1 = 2,\ r_2 = 1,\ r_3 = -1$$

$$y(x) = c_1 e^{2x} + c_2 e^{x} + c_3 e^{-x}$$
$$y'(x) = 2c_1 e^{2x} + c_2 e^{x} - c_3 e^{-x}$$
$$y''(x) = 4c_1 e^{2x} + c_2 e^{x} + c_3 e^{-x}$$

$y(0) = 2 \Rightarrow c_1 + c_2 + c_3 = 2$

$y'(0) = 3 \Rightarrow 2c_1 + c_2 - c_3 = 3$

$y''(0) = 5 \Rightarrow 4c_1 + c_2 + c_3 = 5$

SOLUTION IS

$c_1 = 1,\ c_2 = 1,\ c_3 = 0$

$$\boxed{y(x) = e^{2x} + e^{x}}$$

4. (2 points) Use the Wronskian to show that $y_1(t) = e^{6t}$ and $y_2(t) = e^{-2t}$ are linear independent on $(-\infty, \infty)$.

$$\begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix} = \begin{vmatrix} e^{6t} & e^{-2t} \\ 6e^{6t} & -2e^{-2t} \end{vmatrix} = -2e^{4t} - 6e^{4t}$$
$$= -8e^{4t}$$

NONZERO FOR ANY t

$\Downarrow$

y_1 & y_2 ARE LINEARLY INDEPENDENT.