Math 216 - Quiz 8

April 4, 2012

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2.5 points) Solve: $y'' - 5y' + 4y = 8e^x, \quad y(0) = 5,\ y'(0) = 2$

Homo eq: $y'' - 5y' + 4y = 0$

$r^2 - 5r + 4 = (r-4)(r-1) = 0$

$r = 4,\ r = 1$

$y_h(x) = c_1 e^x + c_2 e^{4x}$

NonHomo Eq: $g(x) = 8e^x$

$y_p(x) = x^s A e^x$

Choose $s = 1$

$y_p(x) = Axe^x$

$y_p'(x) = Ae^x + Axe^x$

$y_p''(x) = 2Ae^x + Axe^x$

$2Ae^x + Axe^x - 5(Ae^x + Axe^x) + 4(Axe^x) = 8e^x$

$\Rightarrow 2A - 5A = 8 \Rightarrow A = -\frac{8}{3}$

$y(x) = c_1 e^x + c_2 e^{4x} - \frac{8}{3} x e^x$

IC's: $y(0) = 5 \Rightarrow c_1 + c_2 = 5$

$y'(x) = c_1 e^x + 4c_2 e^{4x} - \frac{8}{3} e^x - \frac{8}{3} x e^x$

$y'(0) = 2 \Rightarrow c_1 + 4c_2 - \frac{8}{3} = 2$

$c_2 = -\frac{1}{9}, \quad c_1 = \frac{46}{9}$

$\boxed{y(x) = \frac{46}{9} e^x - \frac{1}{9} e^{4x} - \frac{8}{3} x e^x}$

2. (2.5 points) Solve: $y'' - 2y' - 3y = 6x - 2 + 8xe^{5x}$

Homo eq: $y'' - 2y' - 3y = 0$

$r^2 - 2r - 3 = (r-3)(r+1) = 0$

$r = 3,\ r = -1$

$y_h(x) = c_1 e^{3x} + c_2 e^{-x}$

NonHomo eq: $g(x) = 6x - 2 + 8xe^{5x}$

$y_p(x) = Ax + B + Cxe^{5x} + De^{5x}$

$y_p'(x) = A + Ce^{5x} + 5Cxe^{5x} + 5De^{5x}$

$y_p''(x) = 10Ce^{5x} + 25Cxe^{5x} + 25De^{5x}$

$y_p'' - 2y_p' - 3y_p = 6x - 2 + 8xe^{5x}$

$\Rightarrow -3Ax = 6x$

$-2A - 3B = -2$

$10Ce^{5x} + 25De^{5x} - 2Ce^{5x} - 10De^{5x} - 3De^{5x} = 0$

AND

$25Cxe^{5x} - 10Cxe^{5x} - 3Cxe^{5x} = 8xe^{5x}$

$\Downarrow$

$-3A = 6$

$2A + 3B = 2$

$8C + 12D = 0$

$12C = 8$

$A = -2,\ B = 2,$

$C = \frac{2}{3},\ D = -\frac{4}{9}$

$y_p(x) = -2x + 2 + \frac{2}{3} x e^{5x} - \frac{4}{9} e^{5x}$

$\boxed{y(x) = c_1 e^{3x} + c_2 e^{-x} - 2x + 2 + \frac{2}{3} x e^{5x} - \frac{4}{9} e^{5x}}$

3. (2.5 points) Use variation of parameters to solve: $y'' - 4y' + 4y = (x+1)e^{2x}$

Homo Eq: $y'' - 4y' + 4y = 0$

$r^2 - 4r + 4 = (r-2)^2 = 0$

$r = 2$

$y_h(x) = c_1 e^{2x} + c_2 x e^{2x}$

NonHomo Eq: $g(x) = (x+1)e^{2x}$

$y_1(x) = e^{2x}$, $y_2(x) = x e^{2x}$

$$W(x) = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & (2x+1)e^{2x} \end{vmatrix} = e^{4x}$$

$$V_1(x) = \int (-x^2 - x)\,dx = -\frac{x^3}{3} - \frac{x^2}{2}$$

$$V_2(x) = \int (x+1)\,dx = \frac{x^2}{2} + x$$

$$y_p(x) = \left(-\frac{x^3}{3} - \frac{x^2}{2}\right)e^{2x} + \left(\frac{x^2}{2} + x\right)xe^{2x}$$

$$= \left(\frac{x^3}{6} + \frac{x^2}{2}\right)e^{2x}$$

$$y(x) = c_1 e^{2x} + c_2 x e^{2x} + \frac{x^2}{2}e^{2x} + \frac{x^3}{6}e^{2x}$$

4. (2.5 points) Use variation of parameters to solve: $y'' + 9y = \csc 3x$

Homo Eq: $y'' + 9y = 0$

$r^2 + 9 = 0 \Rightarrow r = \pm 3i$

$y_h(x) = c_1 \cos 3x + c_2 \sin 3x$

NonHomo Eq: $g(x) = \csc 3x$

$y_1(x) = \cos 3x$, $y_2(x) = \sin 3x$

$$W(x) = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3$$

$$V_1(x) = \int -\frac{1}{3}\,dx = -\frac{1}{3}x$$

$$V_2(x) = \int \frac{1}{3}\frac{\cos 3x}{\sin 3x}\,dx = \frac{1}{9}\ln|\sin 3x|$$

$$y_p(x) = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \ln|\sin 3x|$$

$$y(x) = c_1 \cos 3x + c_2 \sin 3x - \frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \ln|\sin 3x|$$