Math 216 - Quiz 9 Name Key

April 11, 2012 Score ____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Solve: $x^2y'' - 3xy' + 3y = 2x^4e^x$

Homo eq: $x^2y'' - 3xy' + 3y = 0$

$x = e^t$ gives (Assuming $x > 0$)

$y''(t) - 4y'(t) + 3y(t) = 0$

$r^2 - 4r + 3 = 0$

$(r-3)(r-1) = 0 \Rightarrow r = 3, 1$

$y_h(t) = c_1e^{3t} + c_2e^t$

$y_h(x) = c_1x^3 + c_2x$

NonHomo eq: $y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 2x^2e^x$

$g(x) = 2x^2e^x, \quad y_1(x) = x^3, \quad y_2(x) = x$

$$W(x) = \begin{vmatrix} x^3 & x \\ 3x^2 & 1 \end{vmatrix} = -2x^3$$

$v_1 = \int e^x\,dx = e^x$

$v_2 = \int -x^2e^x\,dx =$

+	$-x^2$	e^x
−	$-2x$	e^x
+	-2	e^x
−	0	e^x

$= -x^2e^x + 2xe^x - 2e^x$

$y_p(x) = x^3e^x + (-x^2e^x + 2xe^x - 2e^x)x$

$= 2x^2e^x - 2xe^x$

$$\boxed{y(x) = c_1x^3 + c_2x + 2x^2e^x - 2xe^x}$$

2. (5 points) Solve: $x^2y'' - xy' + y = \ln x$

Homo eq: $x^2y'' - xy' + y = 0$

$x = e^t$ gives (ASSUMING $x > 0$)

$y''(t) - 2y'(t) + y(t) = 0$.

$r^2 - 2r + 1 = (r-1)^2 = 0$

$r = 1$

$y_h(t) = c_1 e^t + c_2 t e^t$

$y_h(x) = c_1 x + c_2 x \ln x$

NonHomo eq: $y'' - \frac{1}{x}y' + \frac{1}{x^2}y = \frac{\ln x}{x^2}$

$g(x) = \frac{\ln x}{x^2}$, $y_1(x) = x$, $y_2(x) = x \ln x$

$$W(x) = \begin{vmatrix} x & x\ln x \\ 1 & 1 + \ln x \end{vmatrix} = x$$

$$V_1(x) = \int \frac{-(\ln x)^2}{x^2}\,dx = \int -u^2 e^{-u}\,du$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \begin{array}{c|c|c} & & \\ \hline + & -u^2 & e^{-u} \\ - & -2u & -e^{-u} \\ + & -2 & e^{-u} \\ - & 0 & -e^{-u} \end{array}$$

$$= u^2e^{-u} + 2ue^{-u} + 2e^{-u}$$

$$= \frac{(\ln x)^2}{x} + \frac{2\ln x}{x} + \frac{2}{x}$$

$$V_2(x) = \int \frac{\ln x}{x^2}\,dx = \int ue^{-u}\,du$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \begin{array}{c|c|c} & & \\ \hline + & u & e^{-u} \\ - & 1 & -e^{-u} \\ + & 0 & e^{-u} \end{array}$$

$$= -ue^{-u} - e^{-u}$$

$$= -\frac{\ln x}{x} - \frac{1}{x}$$

$$y_p(x) = (\ln x)^2 + 2\ln x + 2 - (\ln x)^2 - \ln x$$

$$\Rightarrow y_p(x) = \ln x + 2$$

$$\boxed{y(x) = c_1 x + c_2 x \ln x + 2 + \ln x}$$