## Undetermined Coefficients for $y'' + c\,y' + d\,y = g(x)$ ($c$ and $d$ are constants)

| | $\boldsymbol{g(x)}$ | $\boldsymbol{y_p(x)}$ |
|---|---|---|
| (1) | $p_n(x) = a_n x^n + \ldots + a_1 x + a_0$ | $x^s P_n(x) = x^s(A_n x^n + \ldots + A_1 x + A_0)$ |
| (2) | $a e^{\alpha x}$ | $x^s A e^{\alpha x}$ |
| (3) | $a\,\cos\beta x + b\,\sin\beta x$ | $x^s(A\,\cos\beta x + B\,\sin\beta x)$ |
| (4) | $p_n(x) e^{\alpha x}$ | $x^s P_n(x) e^{\alpha x}$ |
| (5) | $p_n(x)\cos\beta x + q_m(x)\sin\beta x$, where $q_m(x) = b_m x^m + \ldots + b_1 x + b_0$ | $x^s\{P_N(x)\cos\beta x + Q_N(x)\sin\beta x\}$, where $Q_N(x) = B_n x^N + \ldots + B_1 x + B_0$ and $N = \max(n, m)$ |
| (6) | $a e^{\alpha x}\cos\beta x + b e^{\alpha x}\sin\beta x$ | $x^s(A e^{\alpha x}\cos\beta x + B e^{\alpha x}\sin\beta x)$ |
| (7) | $p_n(x)\,e^{\alpha x}\cos\beta x + q_m(x)\,e^{\alpha x}\sin\beta x$ | $x^s e^{\alpha x}\{P_N(x)\cos\beta x + Q_N(x)\sin\beta x\}$, where $N = \max(n, m)$ |

The nonnegative integer $s$ is chosen to be the least integer such that no term in $y_p(x)$ is a solution of the corresponding homogeneous equation $y'' + c\,y' + d\,y = 0$.

## Variation of Parameters

If $y_1$ are $y_2$ are two linearly independent solutions of $y''+p(x)y'+q(x)y = 0$, then a particular solution of $y'' + p(x)y' + q(x)y = g(x)$ is $y = v_1 y_1 + v_2 y_2$, where

$$v_1(x) = \int \frac{-g(x)y_2(x)}{W[y_1, y_2](x)}\,dx, \qquad v_2(x) = \int \frac{g(x)y_1(x)}{W[y_1, y_2](x)}\,dx,$$

and $W[y_1, y_2](x) = y_1(x)y_2'(x) - y_1'(x)y_2(x)$.

## 2nd Solution from a 1st

If $y_1$ is a nonzero solution of $y'' + p(x)y' + q(x)y = 0$, then $y_2 = v \cdot y_1$, where

$$v(x) = \int \frac{1}{[y_1(x)]^2} \cdot e^{-\int p(x)dx}\,dx,$$

is also solution. Furthermore, $y_1$ and $y_2$ are linearly independent.