Math 233 - Homework 1

September 9, 2021

Name key

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 14.

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1. (2 points) Determine the measure of angle A in triangle ABC, where $A(1,1,8)$, $B(4,-3,-4)$, and $C(-3,1,5)$. Write your final answer in degrees, rounded to the nearest hundredth.

$\vec{AB} = 3\hat{i} - 4\hat{j} - 12\hat{k}$

$\vec{AC} = -4\hat{i} - 3\hat{k}$

$\vec{AB} \cdot \vec{AC} = -12 + 36 = 24$

$\|\vec{AB}\| = \sqrt{9+16+144} = 13$

$\|\vec{AC}\| = \sqrt{16+9} = 5$

$\cos\theta = \frac{24}{(13\times 5)} = \frac{24}{65}$

$\theta = \cos^{-1}\left(\frac{24}{65}\right)$

$\approx$ 68.33°

2. (2 points) Determine the real number α such that $\vec{u} \times \vec{v}$ and $\hat{i}$ are orthogonal, where $\vec{u} = 3\hat{i} + \hat{j} - 5\hat{k}$ and $\vec{v} = 4\hat{i} - 2\hat{j} + \alpha\hat{k}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -5 \\ 4 & -2 & \alpha \end{vmatrix} = \hat{i}(\alpha - 10) - \hat{j}(3\alpha + 20) + \hat{k}(-6-4)$$

$$= (\alpha - 10)\hat{i} + (-3\alpha - 20)\hat{j} - 10\hat{k}$$

$(\vec{u}\times\vec{v}) \cdot \hat{i} = \alpha - 10 = 0$

ORTHOG. $\Longrightarrow$

$\alpha = 10$

Turn over.

3. (2 points) Find the volume of the parallelepiped determined by $\vec{u} = \hat{\imath} + \hat{\jmath}$, $\vec{v} = \hat{\jmath} + \hat{k}$, and $\vec{w} = \hat{\imath} + \hat{k}$.

$$\text{Volume} = \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-1) + 0(0-1)$$

$$= 1 + 1 = \boxed{2}$$

4. (2 points) Find the distance between the point $A(-3, 1, 1)$ and the line with symmetric equations $x = -y = -z$.

Point on line: $B(0,0,0)$

Direction of line: $\vec{v} = \hat{\imath} - \hat{\jmath} - \hat{k}$

$$d = \frac{\|\vec{BA} \times \vec{v}\|}{\|\vec{v}\|}$$

$$\vec{BA} = -3\hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{BA} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{\imath}(-1+1) - \hat{\jmath}(3-1) + \hat{k}(3-1)$$

$$= -2\hat{\jmath} + 2\hat{k}$$

$$d = \frac{\sqrt{4+4}}{\sqrt{1+1+1}} = \boxed{\frac{\sqrt{8}}{\sqrt{3}}}$$

5. (2 points) Find the measure of the acute angle between the planes described by $x + y + z = 0$ and $2x - y + z - 7 = 0$.

$$\vec{n}_1 = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{n}_2 = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\vec{n}_1 \cdot \vec{n}_2 = 2 - 1 + 1 = 2$$

$$\|\vec{n}_1\| = \sqrt{3}$$

$$\|\vec{n}_2\| = \sqrt{6}$$

$$\cos\theta = \frac{2}{\sqrt{18}}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{18}}\right) \approx \boxed{61.87^\circ}$$