Math 233 - Homework 2

September 29, 2021

Name __key__

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due October 5.

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1. (2 points) Parameterize the helix described by $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\hat{k}$ using the arc-length parameter, s, from $t = 0$.

$\vec{r}'(t) = -\sin t\,\hat{i} + \cos t\,\hat{j} + \hat{k}$

$\|\vec{r}'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1}$

$\qquad = \sqrt{2}$

$S = \int_0^t \sqrt{2}\, d\tau = \sqrt{2}\, t$

$S = \sqrt{2}\, t \Rightarrow t = \dfrac{s}{\sqrt{2}}$

$$\boxed{\vec{R}(s) = \cos\left(\tfrac{s}{\sqrt{2}}\right)\hat{i} + \sin\left(\tfrac{s}{\sqrt{2}}\right)\hat{j} + \tfrac{s}{\sqrt{2}}\hat{k}}$$

2. (2 points) Find the principal unit normal vector for $\vec{r}(t) = (t^3 - 4t)\hat{i} + (5t^2 - 2)\hat{j}$ at $t = 0$.

$\vec{r}'(t) = (3t^2 - 4)\hat{i} + (10t)\hat{j}$

$\|\vec{r}'(t)\| = \sqrt{(3t^2-4)^2 + (10t)^2}$

$\qquad = \sqrt{9t^4 + 76t^2 + 16}$

$\hat{T}(t) = \dfrac{(3t^2-4)\hat{i} + (10t)\hat{j}}{\sqrt{9t^4 + 76t^2 + 16}}$

$\hat{T}'(t) = \dfrac{\left(9t^4+76t^2+16\right)^{1/2}\left(6t\,\hat{i} + 10\hat{j}\right) - \left[(3t^2-4)\hat{i} + (10t)\hat{j}\right]\left(\tfrac{1}{2}\right)\left(9t^4+76t^2+16\right)^{-1/2}\left(36t^3 + 152t\right)}{\left(9t^4 + 76t^2 + 16\right)}$

$\hat{T}'(0) = \dfrac{(4)(10\hat{j}) - (-4\hat{i})(\tfrac{1}{2})(\tfrac{1}{4})(0)}{16} = \dfrac{40}{16}\hat{j}$

$\hat{N}(0) = \dfrac{\hat{T}'(0)}{\|\hat{T}'(0)\|} = \dfrac{40/16\,\hat{j}}{40/16} = \hat{j}$

$$\boxed{\hat{N}(0) = \hat{j}}$$

Turn over.

3. (2 points) Find the curvature of the curve described by $y = x - \frac{1}{4}x^2$ at the point where $x = 2$.

$$\frac{dy}{dx} = 1 - \frac{1}{2}x$$

$$K = \frac{|-\frac{1}{2}|}{\sqrt{1 + 0^2}} = |-\frac{1}{2}| = \boxed{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}$$

$$\frac{dy}{dx}\bigg|_{x=2} = 0$$

4. (2 points) A projectile is fired at a height of 1.5 m above the ground with an initial speed of 100 m/s and at an angle of 30° above the horizontal. Determine the range of the projectile.

$$\vec{r}(t) = 100 \cos 30° \, t \, \hat{\imath} + (-4.9t^2 + 100 \sin 30° \, t + 1.5) \hat{\jmath}$$

$$\vec{r}(t) = 50\sqrt{3} \, t \, \hat{\imath} + (-4.9t^2 + 50t + 1.5) \hat{\jmath}$$

RANGE = 1ST COMPONENT WHEN 2ND COMPONENT IS ZERO.

$$-4.9t^2 + 50t + 1.5 = 0$$

$$\Rightarrow t = \frac{-50 - \sqrt{50^2 + 4(4.9)(1.5)}}{-9.8} \approx 10.234 \text{ s}$$

$$(50\sqrt{3})(10.234)$$

$$\approx \boxed{886.3 \text{ m}}$$

5. (2 points) A golf ball is hit in a horizontal direction off the top edge of a building that is 100 ft tall. How fast must the ball be launched to land 450 ft away from the building?

$$\vec{r}(t) = v_0 t \, \hat{\imath} + (-16t^2 + 100) \hat{\jmath}$$

1ST COMP = 450 WHEN 2ND COMP = 0

$$-16t^2 + 100 = 0$$

$$\Downarrow$$

$$t = \frac{5}{2} \text{ s}$$

$$v_0 \left(\frac{5}{2}\right) = 450$$

$$\Downarrow$$

$$\boxed{v_0 = 180 \text{ FT/s}}$$