

Math 233 - Homework 3
October 7, 2021

Name key
Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due October 14.

1. (3 points) Identify the quadric surface.

(a) $4x^2 + y^2 - z^2 = 0$

CONE

(b) $z = 4x^2 - y^2$

Hyperbolic PARABOLOID

(c) $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{12} = 1$

Ellipsoid

(d) $\frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{12} = 1$

Hyperbolid of TWO SHEETS

(e) $z = 4x^2 + 3y^2$

PARABOLOID

(f) $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{12} = 1$

Hyperbolid of ONE SHEET

2. (1 point) Determine the domain of the function $f(x, y) = 4 \ln(y^2 - x)$.

$$y^2 - x > 0$$

$$\{(x, y) \in \mathbb{R}^2 : y^2 > x\}$$

3. (1 point) Determine the range of the function $g(x, y) = \sqrt{16 - 4x^2 - y^2}$.

$$0 \leq z \leq \sqrt{16}$$

Turn over.

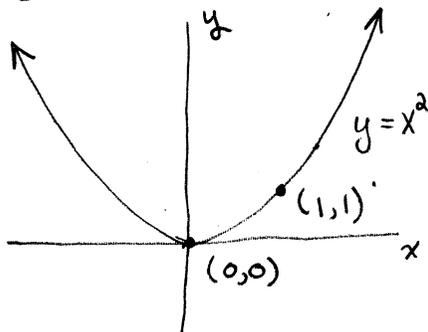
$$\{z \in \mathbb{R} : 0 \leq z \leq 4\}$$

4. (2 points) Let $g(x, y) = \ln\left(\frac{y}{x^2}\right)$. Sketch the level curves $g(x, y) = c$, when $c = 0, 2$.

$$\ln\left(\frac{y}{x^2}\right) = 0$$

$$\downarrow$$

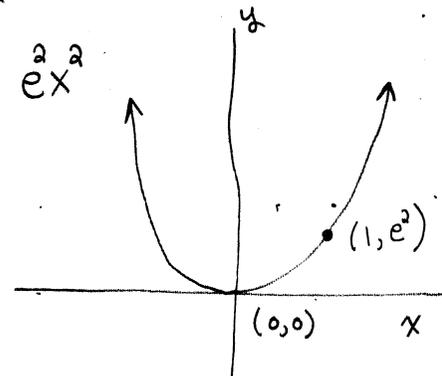
$$y = x^2$$



$$\ln\left(\frac{y}{x^2}\right) = 2$$

$$\frac{y}{x^2} = e^2$$

$$y = e^2 x^2$$



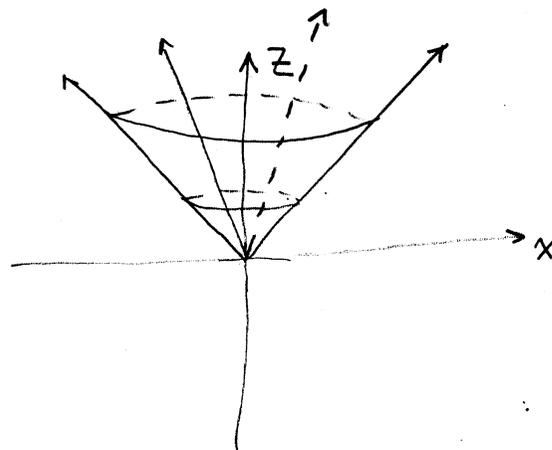
5. (1 point) Sketch the graph of $f(x, y) = \sqrt{x^2 + y^2}$.

$$z = \sqrt{x^2 + y^2}$$

$$z^2 = x^2 + y^2, \quad z \geq 0$$

CONE

UPPER
HALF.



6. (2 points) Evaluate the limit:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}} \cdot \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}}$$

$$= \lim_{(x,y) \rightarrow (0,0)} \frac{x(x-y)(\sqrt{x} + \sqrt{y})}{x-y}$$

$$= 0(\sqrt{0} + \sqrt{0}) = \boxed{0}$$