Math 233 - Homework 4

October 28, 2021

Name key

Score _____

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due November 4.

1. (1 point) Let $z = xe^y$. Use differentials to approximate Δz as (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$.

$$\Delta z \approx \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y = e^y \Delta x + xe^y \Delta y$$

$x=1,\ y=2,\ \Delta x = 0.05,\ \Delta y = 0.1$

$$\Delta z \approx e^2(0.05) + e^2(0.1)$$

$$= 0.15e^2 \approx 1.108358$$

2. (1 point) Find the linearization of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 2, 6)$.

$$f_x(x,y,z) = \frac{x}{\sqrt{x^2+y^2+z^2}} \qquad f_x(3,2,6) = \frac{3}{7} \qquad f(3,2,6) = 7$$

$$f_y(x,y,z) = \frac{y}{\sqrt{x^2+y^2+z^2}} \qquad f_y(3,2,6) = \frac{2}{7}$$

$$f_z(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}} \qquad f_z(3,2,6) = \frac{6}{7}$$

$$L(x,y,z) = 7 + \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$$

3. (2 points) Let $w = xy\cos z$, where $x = t$, $y = t^2$, and $z = \sin^{-1} t$. Use the appropriate chain rule to find $\frac{dw}{dt}$.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= (y\cos z)(1) + (x\cos z)(2t) + (-xy\sin z)\left(\frac{1}{\sqrt{1-t^2}}\right)$$

Turn over.

4. (2 points) Use partial derivatives to find $\frac{dy}{dx}$: $x^2 - 2xy + y^4 = 4$

$f(x,y) = x^2 - 2xy + y^4$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{-(2x-2y)}{-2x+4y^3} = \boxed{\frac{y-x}{2y^3-x}}$$

5. (2 points) Find the directional derivative of $f(x,y,z) = y^2 + xz$ at $(1,2,2)$ in the direction of $\vec{v} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

$\vec{\nabla} f(x,y,z) = z\hat{\imath} + 2y\hat{\jmath} + x\hat{k}$

$\vec{\nabla} f(1,2,2) = 2\hat{\imath} + 4\hat{\jmath} + \hat{k}$

$\vec{u} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\|\vec{u}\| = \sqrt{9} = 3$

$\vec{\nabla} f(1,2,2) \cdot \frac{\vec{u}}{\|\vec{u}\|}$

$= \frac{4-4+2}{3} = \boxed{\frac{2}{3}}$

6. (2 points) Find an equation of the plane tangent to the surface described by $xy + xz + yz = 11$ at the point $(1,2,3)$.

$F(x,y,z) = xy + xz + yz$

$\vec{\nabla} F(x,y,z) = (y+z)\hat{\imath} + (x+z)\hat{\jmath} + (x+y)\hat{k}$

$\vec{n} = \vec{\nabla} F(1,2,3) = 5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$

Tangent plane: $5(x-1) + 4(y-2) + 3(z-3) = 0$

or $5x + 4y + 3z = 22$