Math 233 - Homework 5

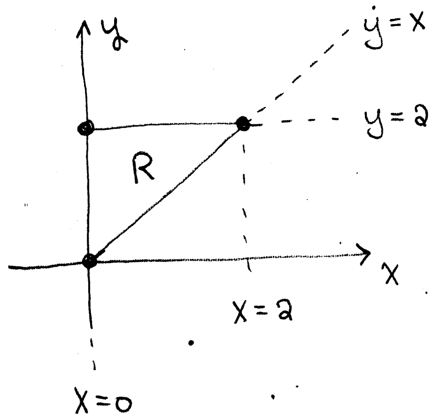
December 2, 2021

Name key

Score

The following problems are from the suggested homework. Show all work to receive full credit. Supply explanations when necessary. This assignment is due December 9.

1. (2.5 points) Evaluate the double integral shown below, where R is the triangle in the xy-plane with vertices at $(0,0)$, $(0,2)$, and $(2,2)$.



$$\iint_R (1-x)\, dA$$

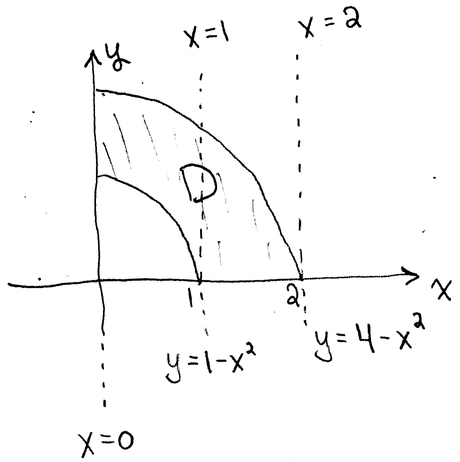
$$= \int_0^2 \int_x^2 (1-x)\, dy\, dx$$

$$= \int_0^2 (1-x)y \Big|_x^2 dx = \int_0^2 (2-2x-x+x^2)\, dx$$

$$= \int_0^2 (x^2-3x+2)\, dx = \frac{1}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2)$$

$$= \frac{2}{3}$$

2. (2.5 points) Let D be the 1st quadrant region bounded by the graphs of $y = 1 - x^2$, $y = 4 - x^2$, $x = 0$, and $y = 0$. Evaluate the double integral.



$$\iint_D x\, dA$$

$$\int_0^1 \int_{1-x^2}^{4-x^2} x\, dy\, dx + \int_1^2 \int_0^{4-x^2} x\, dy\, dx$$

$$= \int_0^1 xy \Big|_{y=1-x^2}^{y=4-x^2} dx + \int_1^2 x(4-x^2)\, dx$$

$$= \int_0^1 3x\, dx + \int_1^2 (4x - x^3)\, dx$$

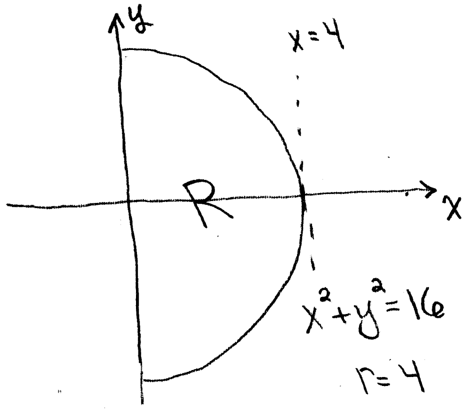
$$= \frac{3}{2}x^2 \Big|_0^1 + \left(2x^2 - \frac{1}{4}x^4\right)\Big|_1^2 = \frac{3}{2} + 8 - 4 - 2 + \frac{1}{4}$$

$$= \frac{15}{4}$$

Turn over.

3. (2.5 points) Evaluate by first converting to polar coordinates.

$$\int_0^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sin(x^2+y^2)\,dy\,dx$$



$$\int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=4} \sin(r^2)\, r\,dr\,d\theta$$

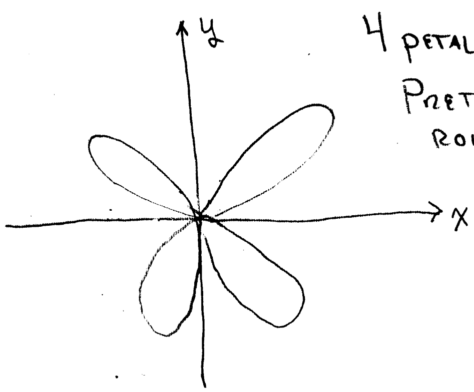
$$= \pi \int_0^4 r \sin r^2\,dr = \frac{\pi}{2}\int_0^{16} \sin u\,du$$

$u = r^2$

$du = 2r\,dr$

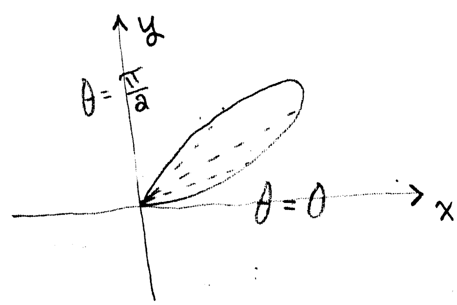
$$= \frac{\pi}{2}\left(-\cos 16 + \cos 0\right) = \boxed{\frac{\pi}{2}\left(1-\cos 16\right)} \approx 3.075$$

4. (2.5 points) Find the total area of the region enclosed by the four-leaved rose $r = \sin 2\theta$.



4 PETAL ROSE.

PRETTY ROUGHLY



$\sin 2\theta = 0$

$\Downarrow$

$\theta = 0$

$\theta = \pi/2$

$$\text{AREA} = 4\int_0^{\pi/2}\int_0^{\sin 2\theta} r\,dr\,d\theta = 4\int_0^{\pi/2} \frac{\sin^2 2\theta}{2}\,d\theta$$

$$= \int_0^{\pi/2} (1-\cos 4\theta)\,d\theta = \theta - \frac{1}{4}\sin 4\theta \Big|_0^{\pi/2}$$

$$= \boxed{\frac{\pi}{2}}$$