

Math 233 - Quiz 1

August 26, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due August 31.

1. (5 points) Consider the points $P(4, -3)$, $Q(9, 2)$, and $R(-3, -5)$.

(a) Find the component form of the vector $2\vec{PQ} - \vec{PR}$.

$$\begin{aligned}\vec{PQ} &= 5\hat{i} + 5\hat{j} \\ \vec{PR} &= -7\hat{i} - 2\hat{j} \\ 2\vec{PQ} - \vec{PR} &= (10\hat{i} + 10\hat{j}) - (-7\hat{i} - 2\hat{j}) \\ &= \boxed{17\hat{i} + 12\hat{j}}\end{aligned}$$

(b) If the initial point of \vec{QR} was placed at the point $(8, 7)$, what would be the terminal point?

$$\begin{aligned}\vec{QR} &= -12\hat{i} - 7\hat{j} \\ (8 + (-12), 7 + (-7)) & \\ &= \boxed{(-4, 0)}\end{aligned}$$

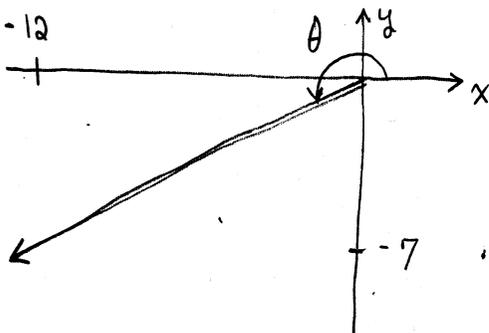
(c) Compute $\|\vec{PQ}\|$.

$$\begin{aligned}\vec{PQ} &= 5\hat{i} + 5\hat{j} \\ \|\vec{PQ}\| &= \sqrt{50} = \boxed{5\sqrt{2}}\end{aligned}$$

(d) Determine the vector (in component form) of magnitude 7 whose direction is opposite that of \vec{PQ} .

$$-\frac{7}{\|\vec{PQ}\|} \vec{PQ} = -\frac{7}{5\sqrt{2}} (5\hat{i} + 5\hat{j}) = \boxed{-\frac{7}{\sqrt{2}}\hat{i} - \frac{7}{\sqrt{2}}\hat{j}}$$

(e) What angle does \vec{QR} make with the positive x -axis? Write your answer in degrees, rounded to the nearest tenth.



$$\tan \theta = \frac{7}{12} \text{ AND } \theta \text{ IN QUAD 3}$$

$$\Rightarrow \theta \approx 30.256^\circ + 180^\circ$$

$$\boxed{\theta \approx 210.3^\circ}$$

Turn over.

2. (2 points) Consider the points $P(3, 1, -2)$ and $Q(9, 2, 0)$ in 3-dimensional space. Let R be the midpoint of the line segment PQ . Determine R and then compute $\|\vec{PR}\|$.

$$R = \left(\frac{3+9}{2}, \frac{1+2}{2}, \frac{-2+0}{2} \right) = \left(6, \frac{3}{2}, -1 \right)$$

$$R \left(6, \frac{3}{2}, -1 \right)$$

$$\vec{PR} = 3\hat{i} + \frac{1}{2}\hat{j} + \hat{k}$$

$$\|\vec{PR}\| = \sqrt{9 + \frac{1}{4} + 1} = \sqrt{10.25} \approx 3.2$$

3. (1 point) The 2-dimensional vector \vec{u} has magnitude 10 and makes a 225° angle with the positive x -axis. Write \vec{u} in terms of \hat{i} and \hat{j} .

$$\vec{u} = 10 \cos 225^\circ \hat{i} + 10 \sin 225^\circ \hat{j}$$

$$= -5\sqrt{2}\hat{i} - 5\sqrt{2}\hat{j}$$

4. (2 points) Determine the angle between $\vec{u} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{w} = -8\hat{i} + 7\hat{k}$. Write your answer in radians, rounded to the nearest hundredth.

$$\vec{u} \cdot \vec{w} = \|\vec{u}\| \|\vec{w}\| \cos \theta$$

$$\vec{u} \cdot \vec{w} = -24 - 35 = -59$$

$$\|\vec{u}\| = \sqrt{9 + 4 + 25} = \sqrt{38}$$

$$\|\vec{w}\| = \sqrt{64 + 49} = \sqrt{113}$$

$$\cos \theta = \frac{-59}{\sqrt{38} \sqrt{113}}$$

$$\theta \approx 2.69$$