**Math 233 - Quiz 2 (IC)**
September 2, 2021

Name key
Score

Show all work to receive full credit. Supply explanations when necessary.

1. (1 point) What does it mean for two vectors to be orthogonal?

TWO VECTORS ARE ORTHOGONAL IF THEIR DOT PRODUCT IS ZERO:

$\vec{u}$ & $\vec{v}$ ARE ORTHOG. IF $\vec{u} \cdot \vec{v} = 0$.

2. (1 point) Let $\vec{u} = 3\hat{\imath} - 5\hat{\jmath} - 2\hat{k}$ and $\vec{w} = \hat{\imath} - 4\hat{\jmath} + 2\hat{k}$. Compute the projection of $\vec{u}$ onto $\vec{w}$.

$$\text{proj}_{\vec{w}} \vec{u} = \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} = \frac{3+20-4}{1+16+4} \vec{w}$$

$$= \frac{19}{21} \vec{w}$$

$$= \frac{19}{21} \hat{\imath} - \frac{76}{21} \hat{\jmath} + \frac{38}{21} \hat{k}$$

3. (1 point) Find the angle between the vectors $\vec{u}$ and $\vec{w}$ in problem #2. Write your final answer in degrees, rounded to the nearest hundredth.

$$\cos\theta = \frac{\vec{u} \cdot \vec{w}}{\|\vec{u}\| \, \|\vec{w}\|} = \frac{19}{\sqrt{38}\sqrt{21}} \Rightarrow \boxed{\theta \approx 47.73^\circ}$$

$\vec{u} \cdot \vec{w} = 19$

$\|\vec{u}\| = \sqrt{9+25+4} = \sqrt{38}$

$\|\vec{w}\| = \sqrt{21}$

# Math 233 - Quiz 2 (TH)

September 2, 2021

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 7.

1. (2 points) Suppose $\vec{u}$ is a nonzero vector such that $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$. Must it be true that $\vec{v} = \vec{w}$? Explain your reasoning.

NO. IF $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$, THEN $\vec{u} \cdot \vec{v} - \vec{u} \cdot \vec{w} = \vec{u} \cdot (\vec{v} - \vec{w})$, AND THIS SIMPLY SAYS $\vec{v} - \vec{w}$ IS ANY VECTOR ORTHOGONAL TO $\vec{u}$. $\vec{v} - \vec{w}$ DOES NOT NEED TO BE THE ZERO VECTOR.

EX $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k}$, BUT $\hat{j} \neq \hat{k}$

2. (2 points) Find the area of the parallelogram determined by the vectors $\vec{u} = 3\hat{\imath} - 5\hat{\jmath} - 2\hat{k}$ and $\vec{w} = \hat{\imath} - 4\hat{\jmath} + 2\hat{k}$.

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & -2 \\ 1 & -4 & 2 \end{vmatrix}$$

$$= \hat{i}(-10-8) - \hat{j}(6+2) + \hat{k}(-12+5)$$

$$= -18\hat{i} - 8\hat{j} - 7\hat{k}$$

AREA $= \|\vec{u} \times \vec{w}\|$

$$= \sqrt{(-18)^2 + (-8)^2 + (-7)^2}$$

$$= \boxed{\sqrt{437} \approx 20.9}$$

3. (3 points) Find a vector of magnitude 5 that is orthogonal to both $\vec{a} = -2\hat{\imath} + 7\hat{k}$ and $\vec{b} = 4\hat{\imath} - 5\hat{\jmath} + \hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 7 \\ 4 & -5 & 1 \end{vmatrix}$$

$$= \hat{i}(0+35) - \hat{j}(-2-28) + \hat{k}(10-0)$$

$$= 35\hat{i} + 30\hat{j} + 10\hat{k}$$

I'LL USE $7\hat{i} + 6\hat{j} + 2\hat{k}$

MAGNITUDE IS $\sqrt{49+36+4} = \sqrt{89}$

VECTOR IS

$$\frac{5}{\sqrt{89}}(7\hat{i} + 6\hat{j} + 2\hat{k})$$