

Math 233 - Quiz 4

October 21, 2021

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 26.

1. (2 points) Let $f(x, y, z) = \sec(x^2y) - \tan(x^3yz^2)$. Find f_x and f_z .

$$f_x(x, y, z) = \sec(x^2y) \tan(x^2y) (2xy) - \sec^2(x^3yz^2) (3x^2yz^2)$$

$$f_z(x, y, z) = -\sec^2(x^3yz^2) (2x^3yz)$$

2. (3 points) Let $g(x, y) = ye^{2x} - \cos(2x - 3y)$. Determine all four second partial derivatives.

$$g_x(x, y) = 2ye^{2x} + 2\sin(2x - 3y)$$

$$g_y(x, y) = e^{2x} - 3\sin(2x - 3y)$$

$$g_{xx}(x, y) = 4ye^{2x} + 4\cos(2x - 3y)$$

$$g_{yy}(x, y) = 9\cos(2x - 3y)$$

$$g_{xy}(x, y) = g_{yx}(x, y) = 2e^{2x} - 6\cos(2x - 3y)$$

Turn over.

$$f_x(x,y) = -y$$

$$f_y(x,y) = 4y - x$$

3. (3 points) Use the definition of differentiability to show that $f(x,y) = 2y^2 - xy$ is differentiable everywhere in \mathbb{R}^2 .

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y) - f(x,y) = 2(y+\Delta y)^2 - (x+\Delta x)(y+\Delta y) - 2y^2 + xy \\ &= \cancel{2y^2} + \underline{4y\Delta y} + 2\Delta y^2 - \cancel{xy} - \underline{x\Delta y} - \underline{y\Delta x} - \underline{\Delta x\Delta y} - \cancel{2y^2} + \cancel{xy} \\ &= \underset{\substack{\uparrow \\ f_x}}{-y\Delta x} + \underset{\substack{\uparrow \\ f_y}}{(4y-x)\Delta y} + \underset{\substack{\uparrow \\ \epsilon_1}}{(-\Delta y)\Delta x} + \underset{\substack{\uparrow \\ \epsilon_2}}{(2\Delta y)\Delta y}\end{aligned}$$

AND $\epsilon_1 \rightarrow 0$ AS $(\Delta x, \Delta y) \rightarrow (0,0)$.
 $\epsilon_2 \rightarrow 0$

ALL OF THE ABOVE IS TRUE FOR ALL (x,y) IN \mathbb{R}^2 .

f IS DIFFERENTIABLE EVERYWHERE.

4. (2 points) Let $F(x,y) = 4y^2 + x^2y - 2xy$. Use differentials to approximate Δz when $(1, -1)$ changes to $(1.03, -1.02)$.

$$dz = (2xy - 2y) dx + (8y + x^2 - 2x) dy$$

$$\Delta z \approx (2xy - 2y) \Delta x + (8y + x^2 - 2x) \Delta y$$

$$x=1, y=-1, \Delta x = 0.03, \Delta y = -0.02$$

$$\Delta z \approx (0)(0.03) + (-9)(-0.02)$$

$$= \boxed{0.18}$$