

# Math 233 - Quiz 5

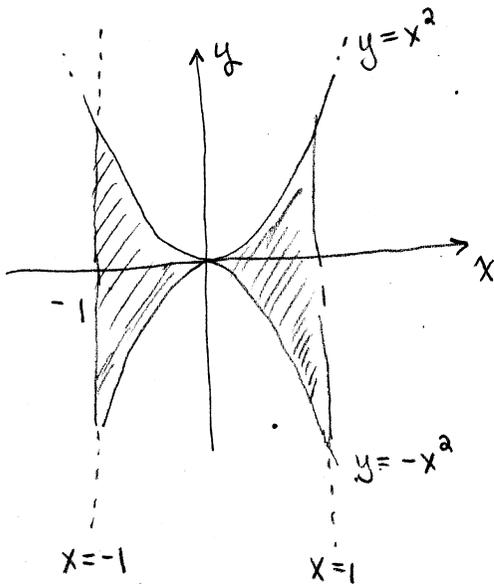
November 18, 2021

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This quiz is due November 30.

1. (3 points) Evaluate the iterated integral and sketch the region of integration.



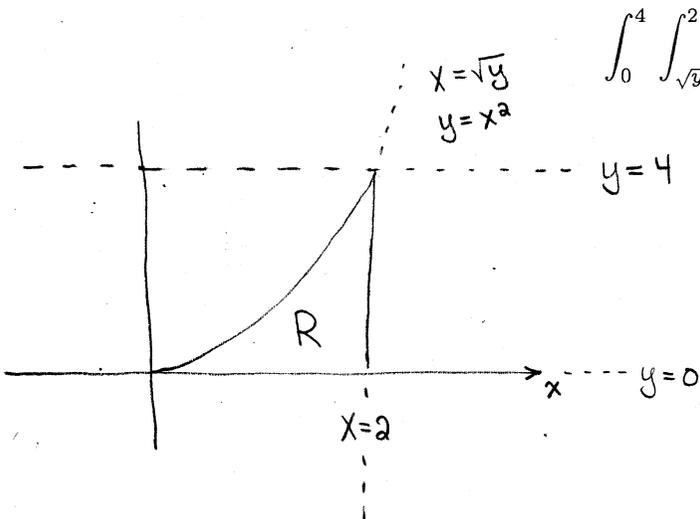
$$\int_{-1}^1 \int_{-x^2}^{x^2} (x^2 - y) dy dx$$

$$= \int_{-1}^1 \left[ yx^2 - \frac{1}{2}y^2 \right]_{-x^2}^{x^2} dx$$

$$= \int_{-1}^1 2x^4 dx = \left. \frac{2}{5}x^5 \right|_{-1}^1$$

$$= \boxed{\frac{4}{5}}$$

2. (3 points) Evaluate the iterated integral by reversing the order of integration.



$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$$

$$= \int_{x=0}^2 \int_{y=0}^{y=x^2} e^{x^3} dy dx$$

$$= \int_0^2 x^2 e^{x^3} dx$$

$$u = x^3 \quad x=0 \Rightarrow u=0$$

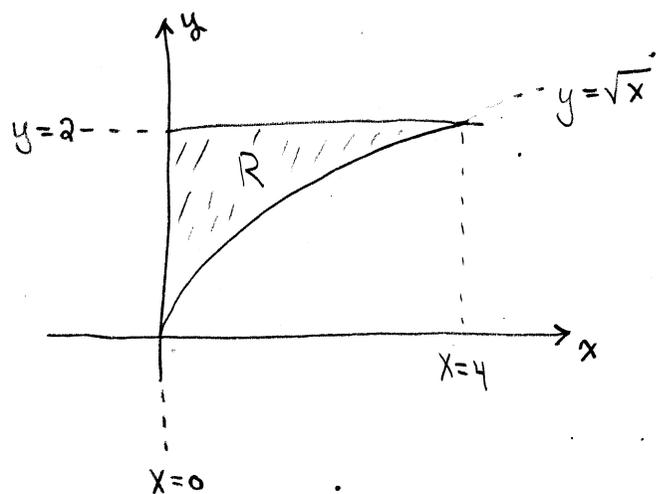
$$du = 3x^2 dx \quad x=2 \Rightarrow u=8$$

$$\frac{1}{3} \int_0^8 e^u du = \frac{1}{3} (e^8 - e^0) = \boxed{\frac{1}{3} (e^8 - 1) \approx 993.32}$$

Turn over.

3. (4 points) Consider the double integral given below, where  $R$  is the plane region bounded by the graphs of  $y = \sqrt{x}$ ,  $y = 2$ , and  $x = 0$ . Sketch the region  $R$ , write the double integral as an iterated integral in both orders, and evaluate either one of your iterated integrals.

$$\iint_R \sin y^3 dA,$$



$$\int_{x=0}^{x=4} \int_{y=\sqrt{x}}^{y=2} \sin y^3 dy dx \quad \text{Type I}$$

$$\int_{y=0}^{y=2} \int_{x=0}^{x=y^2} \sin y^3 dx dy \quad \text{Type II}$$

$$= \int_0^2 y^2 \sin y^3 dy$$

$$u = y^3 \quad y=0 \Rightarrow u=0$$

$$du = 3y^2 dy \quad y=2 \Rightarrow u=8$$

$$= \int_0^8 \frac{1}{3} \sin u du$$

$$= -\frac{1}{3} \cos u \Big|_0^8$$

$$= \frac{1}{3} \cos u \Big|_8^0$$

$$= \frac{1}{3} - \frac{1}{3} \cos 8$$

$$\approx 0.38183$$