Math 233 - Quiz 6

December 7, 2021

Show all work to receive full credit. Supply explanations when necessary. This quiz is due December 16.

1. (5 points) A triangular prism in the 1st octant is bounded by the planes $y = 0$, $y = x$, $x = 1$, $z = 0$, and $z = 1$. The density of the prism at the point (x, y, z) is given by $\rho(x, y, z) = x^3 + y^2 + z + 1$. Find the z-coordinate of the center of mass. Use your calculator or CAS to evaluate the required integrals.
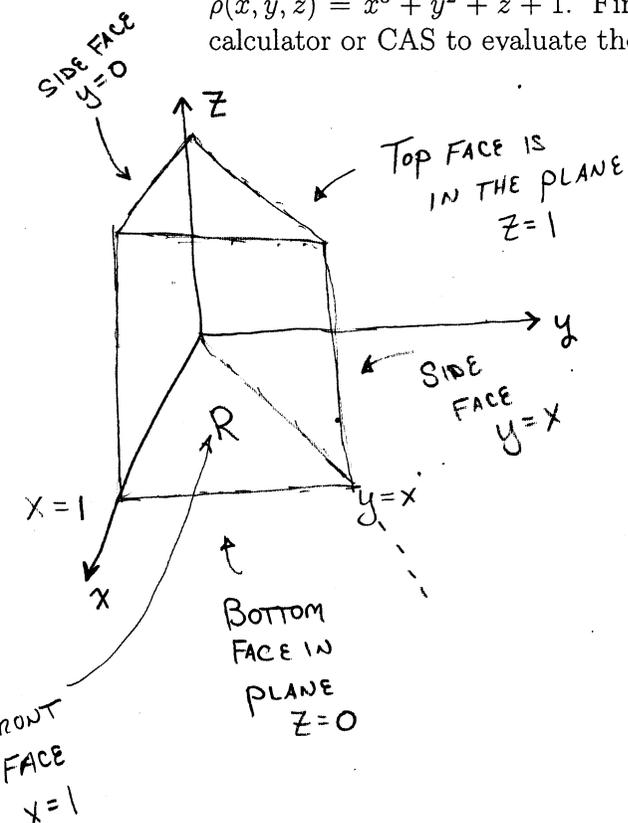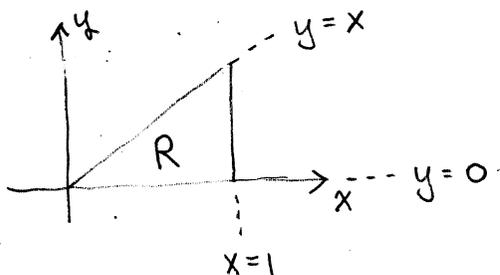

$$M = \int_{x=0}^{x=1} \int_{y=0}^{y=x} \int_{z=0}^{z=1} \left(x^3 + y^2 + z + 1 \right) dz \, dy \, dx$$

$$= \frac{31}{30}$$

$$M_{xy} = \int_{0}^{1} \int_{0}^{x} \int_{0}^{1} z \left(x^3 + y^2 + z + 1 \right) dz \, dy \, dx$$
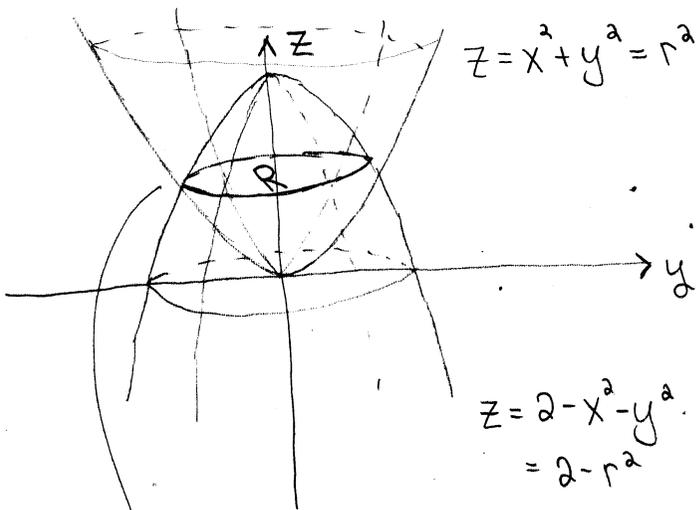
$$= \frac{67}{120}$$

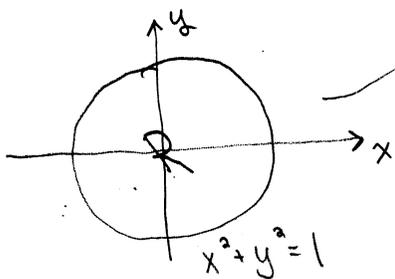$$\boxed{\frac{M_{xy}}{M} = \frac{67}{124}}$$

Turn over.

2. (5 points) Use cylindrical coordinates to find the volume of the solid bounded by the paraboloids $z = 2 - x^2 - y^2$ and $z = x^2 + y^2$. Use your calculator or CAS to evaluate the required integral.

$$z = x^2 + y^2 = r^2$$

$$z = 2 - x^2 - y^2$$
$$= 2 - r^2$$

$$V = \iiint\limits_{S} dV$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} \int_{z=r^2}^{z=2-r^2} r \, dz \, dr \, d\theta$$

$$2 - x^2 - y^2 = x^2 + y^2$$
$$\Downarrow$$
$$x^2 + y^2 = 1$$

$$= \boxed{\pi}$$

$$x^2 + y^2 = 1$$