Math 233 - Test 2

October 14, 2021

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (18 points) In a field goal attempt on a flat field, a football is kicked off the ground at an angle of $30°$ to the horizontal with an initial speed of $65\,\text{ft/sec}$. (Ignore air resistance in this problem and use $g = 32\,\text{ft/s}^2$.)

   (a) What horizontal distance does the football travel while it is in the air?

   (b) Set up the definite integral that gives the length of the path of the football. Use your calculator to estimate the value of the integral.

   (c) To score a field goal, the ball must clear the cross bar of the goal post, which is $10\,\text{ft}$ above the ground. What is the furthest from the goal post the kick can originate and score a field goal?

2. (12 points) An object is moving in such a way that its position at time t is given by

$$\vec{r}(t) = \sin 3t\,\hat{\imath} - \cos 3t\,\hat{\jmath} + 2t^2\hat{k}.$$

(a) Determine the function that gives the speed of the object at time t.

(b) Find the speed at time $t = 5$.

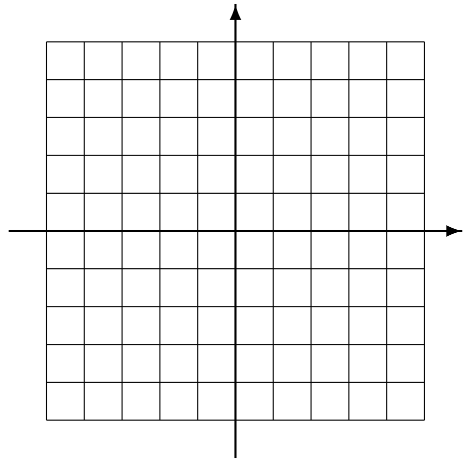
(c) Find $\hat{T}(t)$, the unit tangent vector for $\vec{r}(t)$.

(d) What would you find if you computed $\hat{T}(t) \cdot \hat{T}\,'(t)$? Explain.

3. (8 points) Let $\vec{r}(t) = -\cos 7t\,\hat{\imath} - \sin 7t\,\hat{\jmath} + t\hat{k}$. Compute $\hat{N}(t)$.

4. (8 points) Consider the vector-valued function $\vec{r}(t) = 2t\hat{\imath} - t^2\hat{\jmath}$.

(a) Sketch the graph of $\vec{r}(t)$. Show or describe the orientation of the curve.



(b) Without actually computing $\hat{T}(t)$, sketch the vector $\hat{T}(1)$ on your graph. Label it.

(c) Without actually computing $\hat{N}(t)$, sketch the vector $\hat{N}(1)$ on your graph. Label it.

(d) How would your sketch of $\hat{T}(1)$ be different if the first component of $\vec{r}(t)$ was $-2t$ rather than $2t$? Explain.

5. (3 points) Determine the domain of the function $F(x, y, z) = \ln(1 - x^2 - y^2 - z^2)$.

6. (6 points) For a smooth curve described by $\vec{r}(t)$, the following facts are known:

- $\hat{T}(t_0) = \frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{k}$
- $\hat{T}'(t_0) = -4\hat{\imath} - 3\hat{k}$
- $\|\vec{r'}(t_0)\| = 10$

(a) Find $\vec{r}'(t_0)$.

(b) Find $\hat{N}(t_0)$.

7. (15 points) Determine the limit or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(2,0)} \frac{(x-y)^2 + 2(x-y) - x^2 - 2x}{y}$

(b) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy^3 - 8x^2}{xy^2 - 4}$

(c) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x-y}{\sqrt{x} - \sqrt{y}}$

8. (6 points) Consider the surface described by the equation $4x^2 - y^2 + 9z^2 = 4$.

   (a) Fix a value for one of the variables and draw a good sketch of the corresponding level curve.

   (b) Fix a value for one of the other variables and briefly describe the corresponding level curve.

   (c) Identify the surface.

9. (6 points) Consider the function $h(x, y) = \sqrt{1 + x - y^2}$.

   (a) What is the domain of h?

   (b) Sketch the level curve $h(x, y) = 0$.

   (c) The graph of h is one-half of one of the quadric surfaces that we are familiar with. Describe the graph of h.

10. (8 points) Starting from $t = 0$, compute the arc-length parameter for the curve described by

$$\vec{r}(t) = \frac{1}{2}t^2\,\hat{\imath} + \frac{1}{3}(2t+1)^{3/2}\,\hat{\jmath} + \hat{k}.$$

11. (2 points) Describe the graph of a vector-valued function for which the curvature is 0 for all values of t.

12. (2 points) Describe the graph of a vector-valued function for which the curvature is 1 for all values of t.

13. (6 points) Let a be a positive real number and let $\vec{r}(t) = t\,\hat{\imath} + at^2\,\hat{\jmath}$. The graph of $\vec{r}$ is a parabola. Find the curvature function.