

# Math 233 - Final Exam A

December 9, 2021

Name \_\_\_\_\_

Score \_\_\_\_\_

**Show all work to receive full credit.** Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided. This test is due December 14.

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1. Use the cross product to find the area of  $\triangle ABC$ , where

$$A(1, 2, 1), \quad B(3, -1, 0), \quad C(2, 1, -1).$$

2. Let  $P_1$  be the plane through the points  $A$ ,  $B$ , and  $C$  of the problem above. Let  $P_2$  be the plane described by the equation  $4x + 3y - 2z = 7$ . Find the angle between the planes  $P_1$  and  $P_2$ .

3. In Moscow in 1987, Natalya Lisouskaya set a women's shot put world record by putting a shot 73.833 ft. She launched the shot at a  $40^\circ$  angle to the horizontal from 6.5 ft above the ground. What was the shot's initial speed? (To receive full credit, you must write and use the vector-valued function  $\vec{r}(t)$  that gives the position of the shot at time  $t$ . Also ignore air resistance and use  $g \approx 32 \text{ ft s}^{-2}$ .)

4. Set up the definite integral that gives the length of the space curve

$$\vec{r}(t) = 6t^3\hat{i} - 2t^2\hat{j} + 5t\hat{k}$$

from the point  $(0, 0, 0)$  to the point  $(6, -2, 5)$ . Then use technology to approximate the value of your integral.

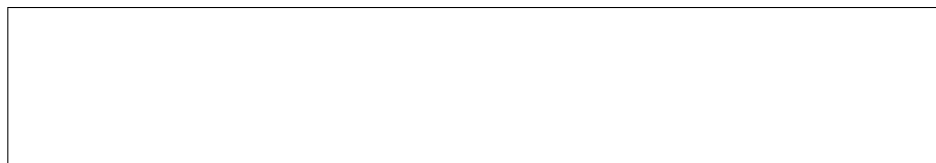
5. Find the limit or show that it does not exist:  $\lim_{(x,y) \rightarrow (1,1)} \frac{xy - y - 2x + 2}{3x - 3}$

6. Let  $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ . Find the linearization of  $f$  at  $(3, 2)$  and use it to approximate  $f(2.99, 1.98)$ .

7. Find and classify the critical points of  $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ .



8. Use a double integral to find the area of the plane region inside the circle  $x^2 + y^2 = 4$ , above the line  $y = 1$ , and below the line  $y = \sqrt{3}x$ . (Hint: Use polar coordinates.)



9. Evaluate the iterated integral by first converting to cylindrical coordinates.

$$\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_0^x (x^2 + y^2) dz dx dy$$

10. A tetrahedron lies in the 1st octant bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$ , and  $2x + y + 4z = 4$ . Its density at the point  $(x, y, z)$  is given by  $\rho(x, y, z) = 1 + x + y^2 + z^3$ . Set up the iterated integral that gives the mass of the solid. Use technology to evaluate the integral.

# Math 233 - Final Exam B

December 16, 2021

Name \_\_\_\_\_

Score \_\_\_\_\_

**Show all work to receive full credit.** Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided.

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1. Determine the measure of angle  $A$  in  $\triangle ABC$ , where

$$A(1, 1, 8), \quad B(4, -3, -4), \quad C(-3, 1, 5).$$

Write your answer in degrees rounded to the nearest hundredth.

2. Find a set of symmetric equations for the line passing through the points  $(1, -2, 3)$  and  $(-3, 1, 1)$ .

3. Let  $\vec{r}(t) = 3 \cos(4t) \hat{i} + 3 \sin(4t) \hat{j} + 5t \hat{k}$ . Find the unit tangent vector at the point where  $t = \pi/2$ .

4. Find the limit or show that it does not exist:  $\lim_{(x,y) \rightarrow (2,1)} \frac{(x-2)(y-1)}{(x-2)^2 + (y-1)^2}$

5. Use differentials to approximate the change in  $z = \sqrt{41 - 4x^2 - y^2}$  as  $(x, y)$  moves from the point  $(2, 3)$  to the point  $(2.1, 2.9)$ .

6. Find an equation of the plane tangent to graph of  $z = 2x^2 - 3xy + 8y^2 + 2x - 4y + 4$  at the point where  $(x, y) = (2, -1)$ .



7. Find the directional derivative of  $f(x, y, z) = \frac{y}{x+z}$  at  $P(2, 1, -1)$  in the direction from  $P$  to  $Q(-1, 2, 0)$ .

8. Let  $D$  be the bounded region between the graphs of  $y = 3 - x^2$  and  $y = 2x$ . Write the double integral as an iterated integral and evaluate.

$$\iint_D x^2 dA$$

9. Sketch the region of integration. Then write the new iterated integral with the reversed order of integration. Do not evaluate the integral.

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy dx$$



10. Evaluate the line integral  $\int_C (x + y + z) ds$ , where  $C$  is the circle described by  $\vec{r}(t) = 2 \cos t \hat{i} + 2 \sin t \hat{k}$  for  $0 \leq t \leq 2\pi$ .

