

Math 233 - Final Exam A

December 9, 2021

Name _____

Score _____

Show all work to receive full credit. Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided. This test is due December 14.

1. Use the cross product to find the area of $\triangle ABC$, where

$$A(1, 2, 1), \quad B(3, -1, 0), \quad C(2, 1, -1).$$

2. Let P_1 be the plane through the points A , B , and C of the problem above. Let P_2 be the plane described by the equation $4x + 3y - 2z = 7$. Find the angle between the planes P_1 and P_2 .

3. In Moscow in 1987, Natalya Lisouskaya set a women's shot put world record by putting a shot 73.833 ft. She launched the shot at a 40° angle to the horizontal from 6.5 ft above the ground. What was the shot's initial speed? (To receive full credit, you must write and use the vector-valued function $\vec{r}(t)$ that gives the position of the shot at time t . Also ignore air resistance and use $g \approx 32 \text{ ft s}^{-2}$.)

4. Set up the definite integral that gives the length of the space curve

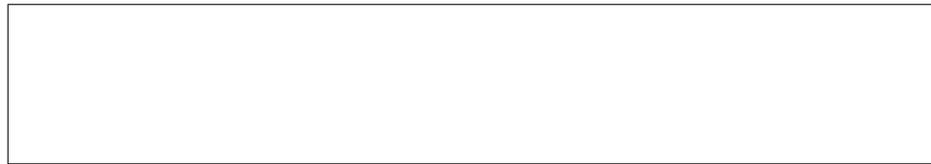
$$\vec{r}(t) = 6t^3\hat{i} - 2t^2\hat{j} + 5t\hat{k}$$

from the point $(0, 0, 0)$ to the point $(6, -2, 5)$. Then use technology to approximate the value of your integral.

5. Find the limit or show that it does not exist: $\lim_{(x,y) \rightarrow (1,1)} \frac{xy - y - 2x + 2}{3x - 3}$

6. Let $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$. Find the linearization of f at $(3, 2)$ and use it to approximate $f(2.99, 1.98)$.

7. Find and classify the critical points of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.



8. Use a double integral to find the area of the plane region inside the circle $x^2 + y^2 = 4$, above the line $y = 1$, and below the line $y = \sqrt{3}x$. (Hint: Use polar coordinates.)



9. Evaluate the iterated integral by first converting to cylindrical coordinates.

$$\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_0^x (x^2 + y^2) dz dx dy$$

10. A tetrahedron lies in the 1st octant bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + y + 4z = 4$. Its density at the point (x, y, z) is given by $\rho(x, y, z) = 1 + x + y^2 + z^3$. Set up the iterated integral that gives the mass of the solid. Use technology to evaluate the integral.

Math 233 - Final Exam B

December 16, 2021

Name _____

Score _____

Show all work to receive full credit. Each problem is worth 5 points—up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided.

1. Determine the measure of angle A in $\triangle ABC$, where

$$A(1, 1, 8), \quad B(4, -3, -4), \quad C(-3, 1, 5).$$

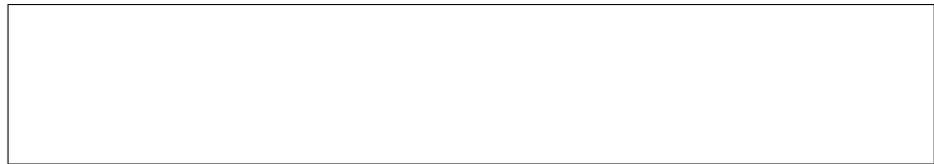
Write your answer in degrees rounded to the nearest hundredth.

2. Find a set of symmetric equations for the line passing through the points $(1, -2, 3)$ and $(-3, 1, 1)$.

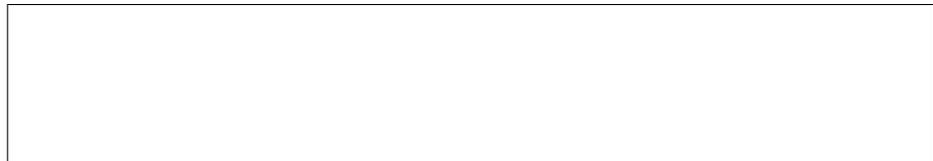
3. Let $\vec{r}(t) = 3 \cos(4t) \hat{i} + 3 \sin(4t) \hat{j} + 5t \hat{k}$. Find the unit tangent vector at the point where $t = \pi/2$.

4. Find the limit or show that it does not exist: $\lim_{(x,y) \rightarrow (2,1)} \frac{(x-2)(y-1)}{(x-2)^2 + (y-1)^2}$

5. Use differentials to approximate the change in $z = \sqrt{41 - 4x^2 - y^2}$ as (x, y) moves from the point $(2, 3)$ to the point $(2.1, 2.9)$.



6. Find an equation of the plane tangent to graph of $z = 2x^2 - 3xy + 8y^2 + 2x - 4y + 4$ at the point where $(x, y) = (2, -1)$.



7. Find the directional derivative of $f(x, y, z) = \frac{y}{x+z}$ at $P(2, 1, -1)$ in the direction from P to $Q(-1, 2, 0)$.

8. Let D be the bounded region between the graphs of $y = 3 - x^2$ and $y = 2x$. Write the double integral as an iterated integral and evaluate.

$$\iint_D x^2 dA$$

9. Sketch the region of integration. Then write the new iterated integral with the reversed order of integration. Do not evaluate the integral.

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy dx$$



10. Evaluate the line integral $\int_C (x + y + z) ds$, where C is the circle described by $\vec{r}(t) = 2 \cos t \hat{i} + 2 \sin t \hat{k}$ for $0 \leq t \leq 2\pi$.

