Math 233 - Final Exam A

Name ____________________________

December 9, 2021

Score ___________

Show all work to receive full credit. Each problem is worth 5 points–up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided. This test is due December 14.

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1. Use the cross product to find the area of $\triangle ABC$, where

$$A\,(1, 2, 1), \quad B\,(3, -1, 0), \quad C\,(2, 1, -1).$$

2. Let P_1 be the plane through the points A, B, and C of the problem above. Let P_2 be the plane described by the equation $4x + 3y - 2z = 7$. Find the angle between the planes P_1 and P_2.

3. In Moscow in 1987, Natalya Lisouskaya set a women's shot put world record by putting a shot 73.833 ft. She launched the shot at a 40° angle to the horizontal from 6.5 ft above the ground. What was the shot's initial speed? (To receive full credit, you must write and use the vector-valued function $\vec{r}(t)$ that gives the position of the shot at time t. Also ignore air resistance and use $g \approx 32\,\text{ft}\,\text{s}^{-2}$.)

4. Set up the definite integral that gives the length of the space curve

$$\vec{r}(t) = 6t^3\hat{\imath} - 2t^2\hat{\jmath} + 5t\hat{k}$$

from the point $(0, 0, 0)$ to the point $(6, -2, 5)$. Then use technology to approximate the value of your integral.

5. Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy - y - 2x + 2}{3x - 3}$

6. Let $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$. Find the linearization of f at $(3, 2)$ and use it to approximate $f(2.99, 1.98)$.

7. Find and classify the critical points of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

8. Use a double integral to find the area of the plane region inside the circle $x^2 + y^2 = 4$, above the line $y = 1$, and below the line $y = \sqrt{3}x$. (Hint: Use polar coordinates.)

9. Evaluate the iterated integral by first converting to cylindrical coordinates.

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy$$

10. A tetrahedron lies in the 1st octant bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x+y+4z = 4$. Its density at the point (x, y, z) is given by $\rho(x, y, z) = 1+x+y^2+z^3$. Set up the iterated integral that gives the mass of the solid. Use technology to evaluate the integral.

Math 233 - Final Exam B

Name ____________________________

December 16, 2021

Score ___________

Show all work to receive full credit. Each problem is worth 5 points–up to 2 points for the answer and up to 3 points for the supporting work or explanation. Place your final answer in the box provided.

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1. Determine the measure of angle A in $\triangle ABC$, where

$$A\,(1,1,8), \quad B\,(4,-3,-4), \quad C\,(-3,1,5).$$

   Write your answer in degrees rounded to the nearest hundredth.

2. Find a set of symmetric equations for the line passing through the points $(1,-2,3)$ and $(-3,1,1)$.

3. Let $\vec{r}(t) = 3\cos(4t)\,\hat{\imath} + 3\sin(4t)\,\hat{\jmath} + 5t\,\hat{k}$. Find the unit tangent vector at the point where $t = \pi/2$.

4. Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(2,1)} \frac{(x-2)(y-1)}{(x-2)^2 + (y-1)^2}$

5. Use differentials to approximate the change in $z = \sqrt{41 - 4x^2 - y^2}$ as (x, y) moves from the point $(2, 3)$ to the point $(2.1, 2.9)$.

6. Find an equation of the plane tangent to graph of $z = 2x^2 - 3xy + 8y^2 + 2x - 4y + 4$ at the point where $(x, y) = (2, -1)$.

7. Find the directional derivative of $f(x, y, z) = \dfrac{y}{x+z}$ at $P\,(2, 1, -1)$ in the direction from P to $Q\,(-1, 2, 0)$.

8. Let D be the bounded region between the graphs of $y = 3 - x^2$ and $y = 2x$. Write the double integral as an iterated integral and evaluate.

$$\iint_D x^2 \, dA$$

9. Sketch the region of integration. Then write the new iterated integral with the reversed order of integration. Do not evaluate the integral.

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx$$

10. Evaluate the line integral $\int_C (x+y+z)\, ds$, where C is the circle described by $\vec{r}(t) = 2\cos t\,\hat{\imath} + 2\sin t\,\hat{k}$ for $0 \le t \le 2\pi$.