Math 233 - Quiz 3

September 8, 2022

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 13.

1. (2 points) Find an equation of the plane containing the points $P(2,4,-1)$, $Q(3,3,8)$, and $R(0,5,-3)$.

$\vec{PQ} = \hat{i} - \hat{j} + 9\hat{k}$

$\vec{PR} = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 9 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(-7) - \hat{j}(16) + \hat{k}(-1)$$

$$= -7\hat{i} - 16\hat{j} - \hat{k}$$

I'll use $\vec{n} = 7\hat{i} + 16\hat{j} + \hat{k}$.

$7x + 16y + z = d$

$7(0) + 16(5) + (-3) = 77$

Plane is

$7x + 16y + z = 77$

2. (2 points) Find the distance from the point $P(9,-4,5)$ to the line described by the symmetric equations

$$4 - x = \frac{y-6}{3} = \frac{z}{5}.$$

$\vec{v} = -\hat{i} + 3\hat{j} + 5\hat{k}$

Point on line: $R(4,6,0)$

$\vec{PR} = -5\hat{i} + 10\hat{j} - 5\hat{k}$

$$\vec{PR} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 10 & -5 \\ -1 & 3 & 5 \end{vmatrix}$$

$$= \hat{i}(65) - \hat{j}(-30) + \hat{k}(-5)$$

$$= 65\hat{i} + 30\hat{j} - 5\hat{k}$$

$\|\vec{PR} \times \vec{v}\| = \sqrt{5150} = 5\sqrt{206}$

$\|\vec{v}\| = \sqrt{35}$

Distance $= \dfrac{5\sqrt{206}}{\sqrt{35}}$

≈ 12.13

Turn over.

3. (2 points) Argue that the planes are parallel. Then find the distance between them.

$$P_1 : 2x - 3y - 7z = 19 \qquad P_2 : -2x + 3y + 7z = 20$$

NORMAL VECTOR FOR $P_1 = \vec{n}_1 = 2\hat{\imath} - 3\hat{\jmath} - 7\hat{k}$

NORMAL VECTOR FOR $P_2 = \vec{n}_2 = -2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

$\vec{n}_1 = -\vec{n}_2 \Rightarrow \vec{n}_1 \parallel \vec{n}_2$

NORMAL VECTORS PARALLEL

$\Rightarrow$ PLANE PARALLEL.

POINT ON P_1: $(5, -3, 0)$

DISTANCE $= \dfrac{|-2(5) + 3(-3) + 7(0) - 20|}{\sqrt{4+9+49}} = \boxed{\dfrac{39}{\sqrt{62}}} \approx 4.95$

4. (2 points) Find a set of symmetric equations for the line of intersection of the planes

$$x + y - 3z = 8 \qquad \text{and} \qquad 3x - 5y + z = 4.$$

NOTICE THAT $(2, 0, -2)$ IS A POINT ON THE LINE OF INTERSECTION.

$\vec{n}_1 = \hat{\imath} + \hat{\jmath} - 3\hat{k}$

$\vec{n}_2 = 3\hat{\imath} - 5\hat{\jmath} + \hat{k}$

$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -3 \\ 3 & -5 & 1 \end{vmatrix} = \hat{\imath}(-14) - \hat{\jmath}(10) + \hat{k}(-8)$

$= -14\hat{\imath} - 10\hat{\jmath} - 8\hat{k}$

I'LL USE $\vec{v} = 7\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$.

$P(2, 0, -2)$

$$\boxed{\text{LINE: } \frac{x-2}{7} = \frac{y}{5} = \frac{z+2}{4}}$$

5. (2 points) Find the angle between the planes in problem 4. Write your final answer in degrees, rounded to the nearest hundredth.

$\vec{n}_1 = \hat{\imath} + \hat{\jmath} - 3\hat{k}$

$\vec{n}_2 = 3\hat{\imath} - 5\hat{\jmath} + \hat{k}$

$\vec{n}_1 \cdot \vec{n}_2 = 3 - 5 - 3 = -5$

$\|\vec{n}_1\| = \sqrt{11}$

$\|\vec{n}_2\| = \sqrt{35}$

$\cos\theta = \dfrac{5}{\sqrt{11}\sqrt{35}} \approx 0.2548$

$$\boxed{\theta \approx 75.24^\circ}$$