# Math 233 - Quiz 4

September 22, 2022

Name Key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 27.

1. (3 points) Find a vector-valued function $\vec{r}(t)$ that satisfies

$$\vec{r}\,'(t) = 4\sin(2t)\,\hat{\imath} + (te^{t^2} + t)\,\hat{\jmath} + \frac{5}{t+1}\,\hat{k}, \qquad \vec{r}(0) = \hat{\imath} + 2\hat{\jmath} - \hat{k}.$$

$$\int te^{t^2}\,dt = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^{t^2}$$

$u = t^2$

$du = 2t\,dt$

$$\vec{r}(t) = \left[-2\cos(2t) + c_1\right]\hat{\imath} + \left[\frac{1}{2}e^{t^2} + \frac{1}{2}t^2 + c_2\right]\hat{\jmath} + \left[5\ln|t+1| + c_3\right]\hat{k}$$

$$\vec{r}(0) = (-2 + c_1)\hat{\imath} + \left(\frac{1}{2} + c_2\right)\hat{\jmath} + c_3\hat{k} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$$

$$\Rightarrow c_1 = 3,\ c_2 = \frac{3}{2},\ c_3 = -1$$

$$\vec{r}(t) = \left[-2\cos(2t) + 3\right]\hat{\imath} + \left[\frac{1}{2}e^{t^2} + \frac{1}{2}t^2 + \frac{3}{2}\right]\hat{\jmath} + \left[5\ln|t+1| - 1\right]\hat{k}$$

2. (2 points) Let $\vec{r}(t) = \ln(t^2+1)\,\hat{\imath} + \sin(\pi t)\,\hat{\jmath} + e^{1-t}\,\hat{k}$. Compute $\hat{T}(1)$.

$$\vec{r}\,'(t) = \frac{2t}{t^2+1}\hat{\imath} + \pi\cos(\pi t)\hat{\jmath} - e^{1-t}\hat{k}$$

$$\vec{r}\,'(1) = \hat{\imath} - \pi\hat{\jmath} - \hat{k}$$

$$\|\vec{r}\,'(1)\| = \sqrt{1 + \pi^2 + 1} = \sqrt{2 + \pi^2}$$

$$\hat{T}(1) = \frac{\hat{\imath} - \pi\hat{\jmath} - \hat{k}}{\sqrt{2+\pi^2}}$$

*Turn over.*

3. (2 points) Reparameterize the position vector in terms of the arc-length parameter.

$$\vec{r}(t) = (2t-4)\,\hat{\imath} + (3t-9)\,\hat{\jmath} - (8t-1)\,\hat{k}$$

$\vec{r}\,'(t) = 2\hat{\imath} + 3\hat{\jmath} - 8\hat{k}$

$\|\vec{r}\,'(t)\| = \sqrt{4+9+64} = \sqrt{77}$.

$S = \int_0^t \sqrt{77}\,dt = \sqrt{77}\,t$

Assuming we start at $t=0$.

$t = \frac{S}{\sqrt{77}} \Rightarrow \boxed{\vec{R}(s) = \left(\frac{2s}{\sqrt{77}} - 4\right)\hat{\imath} + \left(\frac{3s}{\sqrt{77}} - 9\right)\hat{\jmath} - \left(\frac{8s}{\sqrt{77}} - 1\right)\hat{k}}$

4. (3 points) Find the curvature at the point $P$.

$$\vec{r}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath} + \frac{t^3}{4}\,\hat{k}, \quad P\,(2,4,2) \Rightarrow t = 2$$

$\vec{r}\,'(t) = \hat{\imath} + 2t\,\hat{\jmath} + \frac{3}{4}t^2\,\hat{k}$

$\vec{r}\,''(t) = 2\hat{\jmath} + \frac{3}{2}t\,\hat{k}$

$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2t & \frac{3}{4}t^2 \\ 0 & 2 & \frac{3}{2}t \end{vmatrix}$

$= \left(3t^2 - \frac{3}{2}t^2\right)\hat{\imath} - \left(\frac{3}{2}t\right)\hat{\jmath} + (2)\hat{k}$

$= \frac{3}{2}t^2\,\hat{\imath} - \frac{3}{2}t\,\hat{\jmath} + 2\hat{k}$

$\|\vec{r}\,' \times \vec{r}\,''\| = \sqrt{\frac{9}{4}t^4 + \frac{9}{4}t^2 + 4}$

$\|\vec{r}\,'\| = \sqrt{1 + 4t^2 + \frac{9}{16}t^4}$

$K(t) = \dfrac{\left(\frac{9}{4}t^4 + \frac{9}{4}t^2 + 4\right)^{1/2}}{\left(1 + 4t^2 + \frac{9}{16}t^4\right)^{3/2}}$

$K(2) = \dfrac{(36+9+4)^{1/2}}{(1+16+9)^{3/2}}$

$= \dfrac{\sqrt{49}}{26^{3/2}} = \boxed{\dfrac{7}{26^{3/2}} \approx 0.0528}$