# Math 233 - Quiz 5

October 6, 2022

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 11.

1. (1 point) Assume $k$ is a positive constant and compute the curvature of the graph of $y = e^{kx}$ at the point where $x = 0$.

$$\frac{dy}{dx} = k e^{kx}$$

$$\frac{d^2y}{dx^2} = k^2 e^{kx}$$

$$K(x) = \frac{k^2 e^{kx}}{\left(1 + k^2 e^{2kx}\right)^{3/2}}$$

$$K(0) = \frac{k^2}{\left(1 + k^2\right)^{3/2}}$$

2. (3 points) Let $\vec{r}(t) = t\,\hat{\imath} + \ln(\cos t)\,\hat{\jmath} + 5\,\hat{k}$. Compute $\hat{T}(t)$ and $\hat{N}(t)$.

$$\vec{r}'(t) = \hat{\imath} - \tan t\,\hat{\jmath}$$

WE MUST ASSUME $\cos t > 0$

$$\|\vec{r}'(t)\| = \sqrt{1 + \tan^2 t} = \sqrt{\sec^2 t} = |\sec t| = \sec t$$

$$\hat{T}(t) = \frac{\hat{\imath} - \tan t\,\hat{\jmath}}{\sec t} = \cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$$

$$\hat{N}(t) = -\sin t\,\hat{\imath} - \cos t\,\hat{\jmath}$$

*Turn over.*

$$\vec{r}(t) = 80t\,\hat{\imath} + (-16t^2 + 80t + 3)\,\hat{\jmath}$$

3. (6 points) A baseball is hit from 3 ft above home plate with an initial velocity vector of $\vec{v}(0) = \langle 80, 80 \rangle$. Assume the playing field is flat, ignore all forces except gravity, and use $g = 32\,\text{ft/sec}^2$.

(a) How far does the ball travel horizontally?

$$-16t^2 + 80t + 3 = 0 \implies t = \frac{-80 - \sqrt{80^2 - 4(-16)(3)}}{-32} \approx 5.0372 \text{ sec}$$

$$80\,(5.0372) \approx \boxed{403 \text{ FT}}$$

(b) What is the maximum height of the ball?

$$-32t + 80 = 0 \implies t = \frac{80}{32} = \frac{5}{2}$$

$$-16\left(\frac{25}{4}\right) + 80\left(\frac{5}{2}\right) + 3 = \boxed{103 \text{ FT}}$$

(c) What is the length of the baseball's entire path? (Use technology to approximate the value of your integral.)

$$\vec{r}'(t) = 80\,\hat{\imath} + (-32t + 80)\,\hat{\jmath}$$

$$\int_0^{5.0372} \sqrt{80^2 + (-32t + 80)^2}\; dt \approx \boxed{463.3 \text{ FT}}$$

(d) Does the ball clear a 20-ft fence that is 380 ft downrange?

$$80t = 380$$

$$\implies t = \frac{380}{80} = \frac{19}{4}$$

$$-16\left(\frac{19}{4}\right)^2 + 80\left(\frac{19}{4}\right) + 3 = 22 \text{ FT}$$

YES, IT CLEARS BY 2 FT.