Math 233 - Quiz 6

October 20, 2022

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 25.

1. (4 points) Determine the limit or show that it does not exist.

(a) $\lim\limits_{(x,y)\to(3,1)} \dfrac{3y - x + xy - 3}{y - 1}$ 0/0 More work

$$\lim_{(x,y)\to(3,1)} \frac{\cancel{(y-1)}(x+3)}{\cancel{(y-1)}} = \boxed{6}$$

(b) $\lim\limits_{(x,y)\to(2,2)} \dfrac{x^2y^2 - 8x}{x^2y - 8}$ 0/0

Along $x = 2$:

$$\lim_{y\to 2} \frac{4y^2 - 16}{4y - 8} = \lim_{y\to 2} \frac{4(y+2)(y-2)}{4(y-2)} = 4$$

Along $y = x$:

$$\lim_{x\to 2} \frac{x^4 - 8x}{x^3 - 8} = \lim_{x\to 2} \frac{x\cancel{(x^3-8)}}{\cancel{x^3-8}} = 2$$

Limit DNE by two-path test.

Turn over.

2. (2 points) Let $g(x,y) = \ln(x\sqrt{x^2+y^4})$. Find g_x and g_y.

$$g(x,y) = \ln x + \frac{1}{2}\ln(x^2+y^4)$$

$$g_x(x,y) = \frac{1}{x} + \frac{1}{2}\,\frac{2x}{x^2+y^4} = \boxed{\frac{1}{x} + \frac{x}{x^2+y^4}}$$

$$g_y(x,y) = \frac{1}{2}\,\frac{4y^3}{x^2+y^4} = \boxed{\frac{2y^3}{x^2+y^4}}$$

3. (2 points) Let $f(x,y,z) = e^{-x}\sin(yz)$. Find the mixed partial derivative f_{yzy}.

$$f_y(x,y,z) = ze^{-x}\cos(yz)$$

It will be equal to f_{yyz}

$$f_{yy}(x,y,z) = -z^2e^{-x}\sin(yz)$$

$$f_{yyz}(x,y,z) = -2ze^{-x}\sin(yz) - yz^2e^{-x}\cos(yz)$$

$$= f_{yzy}(x,y,z)$$

4. (2 points) Let $w = 3xy^2z^3 + e^{-x^2y}$. Use differentials to approximate Δw as (x,y,z) changes from $(0,1,2)$ to $(0.01, 0.97, 2.02)$.

$$\frac{\partial w}{\partial x} = 3y^2z^3 - 2xye^{-x^2y} \qquad \left.\frac{\partial w}{\partial x}\right|_{(0,1,2)} = 24$$

$$\Delta x = 0.01 - 0 = 0.01$$

$$\frac{\partial w}{\partial y} = 6xyz^3 - x^2e^{-x^2y} \qquad \left.\frac{\partial w}{\partial y}\right|_{(0,1,2)} = 0$$

$$\Delta w \approx 24(0.01)$$

$$= \boxed{0.24}$$

$$\frac{\partial w}{\partial z} = 9xy^2z^2 \qquad \left.\frac{\partial w}{\partial z}\right|_{(0,1,2)} = 0$$

$$\Delta w \approx \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z$$