Math 233 - Quiz 8

November 3, 2022

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due November 8.

1. (3 points) Suppose that z is implicitly defined as a function of x and y by the equation

$$\frac{xyz}{yz + xz + xy} = 1.$$

Find $\partial z/\partial x$ and $\partial z/\partial y$.

EASIER TO SAY $xyz = yz + xz + xy$ FOR $yz + xz + xy \neq 0$.

$F(x,y,z) = xyz - yz - xz - xy$

OR TO SIMPLIFY...

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-(yz - z - y)}{xy - y - x} \cdot \frac{xz}{xz} = \frac{-z(xyz - xz - xy)}{x(xyz - yz - xz)} = \frac{-zyz}{xxy} = \frac{-z^2}{x^2}$$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-(xz - z - x)}{xy - y - x} \cdot \frac{yz}{yz} = \frac{-z(xyz - yz - xy)}{y(xyz - yz - xz)} = \frac{-zxz}{yxy} = \frac{-z^2}{y^2}$$

2. (2 points) Find the directional derivative of $f(x, y) = \ln(x^2 + y^2)$ at $(1, 2)$ in the direction of $\vec{v} = -3\hat{\imath} + 4\hat{\jmath}$.

$$\vec{\nabla} f(x,y) = \frac{2x}{x^2+y^2}\hat{\imath} + \frac{2y}{x^2+y^2}\hat{\jmath}$$

$$\vec{\nabla} f(1,2) = \frac{2}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$$

$\|\vec{v}\| = 5$

$$\frac{\vec{\nabla} f(1,2)\cdot\vec{v}}{\|\vec{v}\|} = \frac{\frac{2}{5}(-3) + \left(\frac{4}{5}\right)(4)}{5}$$

$$= \frac{10}{25} = \boxed{\frac{2}{5}}$$

Turn over.

3. (2.5 points) Find an equation of the plane tangent to the graph of the equation $xe^y\cos(z) - z = 1$ at the point $(1, 0, 0)$.

OUR SURFACE IS THE LEVEL SURFACE $F(x,y,z) = 1$

WHERE $F(x,y,z) = xe^y\cos(z) - z$

$$\vec{\nabla} F(x,y,z) = e^y \cos z\ \hat{i} + xe^y \cos z\ \hat{j} + (-xe^y \sin z - 1)\ \hat{k}$$

$$\vec{n} = \vec{\nabla} F(1,0,0) = \hat{i} + \hat{j} - \hat{k}$$

TANGENT PLANE IS ...

$$(x-1) + y - z = 0$$

OR

$$x + y - z = 1$$

4. (2.5 points) The electrical potential in a certain region of space is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

Find the $\underbrace{\text{maximum value of the directional derivative}}_{\text{MAGNITUDE OF GRADIENT.}}$ of V at the point $(3, 4, 5)$.

$$\vec{\nabla} V(x,y,z) = (10x - 3y + yz)\hat{i} + (-3x + xz)\hat{j} + (xy)\hat{k}$$

$$\vec{\nabla} V(3,4,5) = 38\hat{i} + 6\hat{j} + 12\hat{k}$$

$$\|\vec{\nabla} V(3,4,5)\| = \sqrt{38^2 + 6^2 + 12^2} = \boxed{\sqrt{1624} \approx 40.299}$$