Math 233 - Quiz 9

November 17, 2022
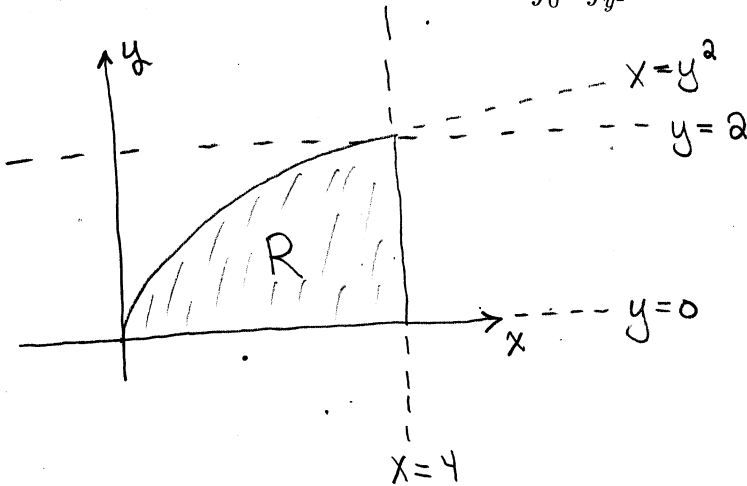
Name ___Key___

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due November 29.

1. (5 points) Sketch the region of integration, reverse the order of integration, and evaluate.

$$\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x \, dx \, dy$$

$$\int_0^4 \int_0^{\sqrt{x}} \sqrt{x}\sin x \, dy \, dx$$

$$= \int_0^4 y\sqrt{x}\sin x \Big|_{y=0}^{y=\sqrt{x}} dx$$

$$= \int_0^4 x\sin x \, dx$$

$$= \left(-x\cos x + \sin x\right)\Big|_{x=0}^{x=4}$$

INT BY PARTS
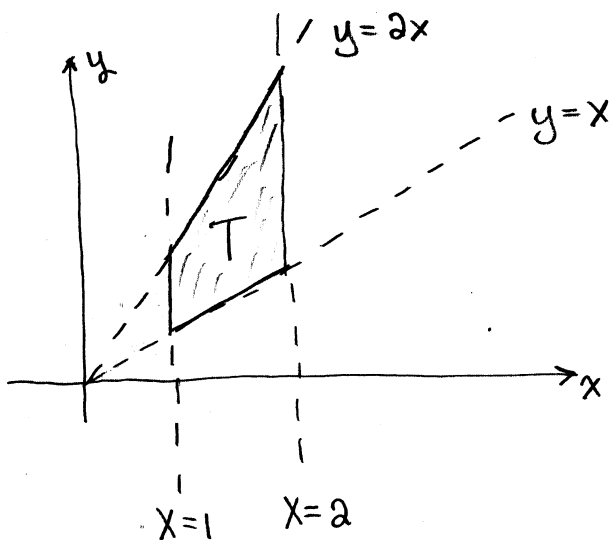
+	x	$\sin x$
−	1	$-\cos x$
+	0	$-\sin x$

$$= \boxed{\sin(4) - 4\cos(4)}$$

$$\approx 1.85777$$

Turn over.

2. (5 points) Let T to the trapezoid bounded by the graphs of $y = x$, $y = 2x$, $x = 1$, and $x = 2$. Find the average value of $f(x,y) = \dfrac{y}{x^2 + y^2}$ over T.

$| / \quad y = 2x$

$\uparrow y$

$- y = x$

T

$X = 1 \qquad X = 2$

$\rightarrow x$

AREA OF T

$$= \int_1^2 (2x - x)\, dx = \int_1^2 x\, dx$$

$$= \frac{1}{2}x^2 \Big|_1^2 = \frac{3}{2}$$

AVERAGE VALUE $= \dfrac{2}{3} \displaystyle\iint_T \dfrac{y}{x^2 + y^2}\, dA$

$$= \frac{2}{3} \int_1^2 \int_x^{2x} \frac{y}{x^2 + y^2}\, dy\, dx = \frac{1}{3} \int_1^2 \int_{2x^2}^{5x^2} \frac{du}{u}$$

$$u = x^2 + y^2$$
$$du = 2y\, dy$$

$$= \frac{1}{3} \int_1^2 \ln 5x^2 - \ln 2x^2 \, dx$$

$$= \frac{1}{3} \int_1^2 \ln\left(\frac{5}{2}\right) dx$$

$$= \boxed{\frac{1}{3} \ln\left(\frac{5}{2}\right)}$$