Math 233 - Test 1

September 15, 2022

Name ______________________

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (9 points) In the following problems, the vectors $\vec{u}$ and $\vec{w}$ are 2D vectors in the xy-plane.

   (a) The vector $\vec{w}$ has magnitude 5 and makes a $120°$ angle with the positive x-axis. Find the component form of $\vec{w}$.

   (b) The vector $\vec{u} + \vec{w}$ has component form $-3\,\hat{\imath} + \sqrt{3}\,\hat{\jmath}$. Find the component form of $\vec{u}$.

   (c) What angle does $\vec{u}$ make with the positive x-axis?

2. (5 points) Find a vector of magnitude 5 that is parallel to the line with symmetric equations

$$\frac{5-x}{3} = \frac{y-6}{4} = z.$$

3. (8 points) Find the acute angle between the planes described by the equations below. Write your final answer in degrees, rounded to the nearest hundredth.

$$3x + 4y - 9z = 1 \qquad\qquad 2y + 8z = 5$$

4. (12 points) Consider the plane described by the equation $2x - y + 6z = 7$.

   (a) Find a point on the plane. Call it P, and show (or explain) how you know P is on the plane.

   (b) Let $Q\,(4, -3, 2)$. Show that Q is NOT on the plane.

   (c) Let $\vec{n}$ be a vector normal to the plane. Compute $\operatorname{proj}_{\vec{n}} \vec{PQ}$.

   (d) Compute $\|\operatorname{proj}_{\vec{n}} \vec{PQ}\|$. (You have just computed the distance from Q to the plane.)

5. (10 points) A triangle has vertices at the points $A\,(1, 3, -2)$, $B\,(1, 1, -5)$, and $C\,(8, 0, -3)$.

   (a) Find the area of $\triangle ABC$.

   (b) Find an equation of the plane containing A, B, and C.

6. (3 points) Sketch two vectors that share a common initial point. Label them $\vec{a}$ and $\vec{b}$. Then sketch and label the vector $\text{proj}_{\vec{b}}\,\vec{a}$.

7. (6 points) Show that the points are collinear. Explain your reasoning.

$$P\,(2, -1, 7), \qquad Q\,(17, -10, 10), \qquad R\,(-8, 5, 5)$$

8. (6 points) What does it mean for two vectors to be orthogonal? Give an example of two nonzero, orthogonal vectors in 3D-space, and show that your vectors are orthogonal.

9. (6 points) Find a set of parametric equations for the line **segment** connecting $R(3,-4,-2)$ and $S(4,3,9)$.

10. (4 points) Suppose you were given two nonzero vectors $\vec{u}$ and $\vec{v}$ in 3D-space. Explain how you could find a nonzero vector that is orthogonal to both $\vec{u}$ and $\vec{v}$.

11. (6 points) Suppose you were given two nonzero vectors in 3D-space. Briefly describe two different ways that you could test whether the vectors are parallel.

12. (8 points) Plot four or five points on the graph of $\vec{r}(t) = (1+t)\,\hat{\imath} + (1-t^2)\,\hat{\jmath}$. Then sketch the graph. Draw arrows on your graph to indicate the curve's orientation.

13. (3 points) Look back at the problem above. Now describe the graph of $\vec{R}(t) = (1+t)\,\hat{\imath} + (1-t^2)\,\hat{\jmath} + t\,\hat{k}$.

14. (6 points) Let $\vec{r}(t) = \sqrt{t-1}\,\hat{\imath} + \dfrac{\ln t}{t-5}\,\hat{\jmath} + \cos(\pi t)\,\hat{k}$.

   (a) Determine the domain of $\vec{r}$.

   (b) Compute $\lim_{t \to 17} \vec{r}(t)$.

15. (8 points) Let $\vec{r}(t) = \sin 6t\,\hat{\imath} + \cos 6t\,\hat{\jmath} + 8t\,\hat{k}$.

(a) Let $\hat{T}(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$. Compute $\hat{T}(t)$.

(b) Compute $\hat{T}(t) \cdot \hat{T}\,'(t)$.