

Math 233 - Quiz 10

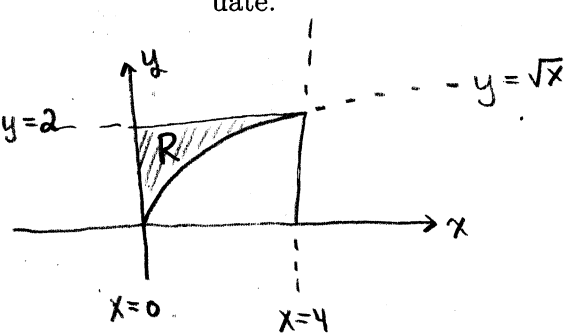
November 16, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Sketch the region of integration, reverse the order of integration, and evaluate.



$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{2+y^3} dy dx$$

$$\int_{y=0}^2 \int_{x=0}^{x=y^2} \frac{3}{2+y^3} dx dy = \int_0^2 \frac{3x}{2+y^3} \Big|_{x=0}^{x=y^2} dy$$

$$= \int_0^2 \frac{3y^2}{2+y^3} du = \int_{u=2}^{u=10} \frac{1}{u} du$$

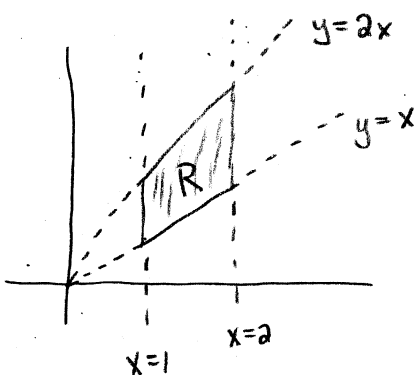
$u = 2+y^3$
 $du = 3y^2 dy$

$$= \ln 10 - \ln 2 = \boxed{\ln 5}$$

2. (5 points) Evaluate the double integral

$$\iint_T \frac{y}{x^2+y^2} dA,$$

where T is the trapezoid bounded by the graphs of $y = x$, $y = 2x$, $x = 1$, and $x = 2$.



$$\int_{x=1}^2 \int_{y=x}^{y=2x} \frac{y}{x^2+y^2} dy dx = \int_1^2 \frac{1}{2} \ln(x^2+y^2) \Big|_{y=x}^{y=2x} dx$$

$$= \int_1^2 \left[\frac{1}{2} \ln(5x^2) - \frac{1}{2} \ln(2x^2) \right] dx = \frac{1}{2} \int_1^2 \ln\left(\frac{5}{2}\right) dx$$

$$= \frac{1}{2} \ln\left(\frac{5}{2}\right) \times \Big|_1^2 = \boxed{\frac{1}{2} \ln\left(\frac{5}{2}\right)}$$