

Math 233 - Quiz 11

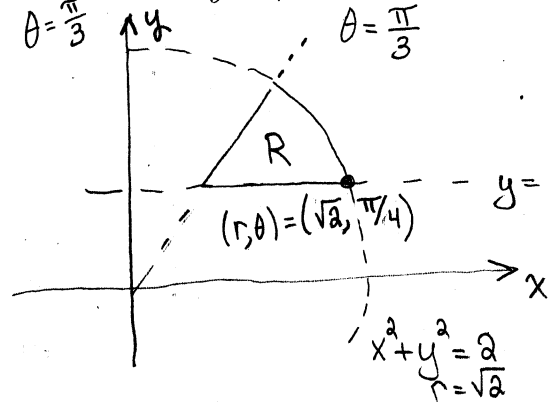
November 30, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Use a double integral in polar coordinates to find the area of the region in the xy -plane inside the circle $x^2 + y^2 = 2$, above the line $y = 1$, and below the line $y = \sqrt{3}x$.



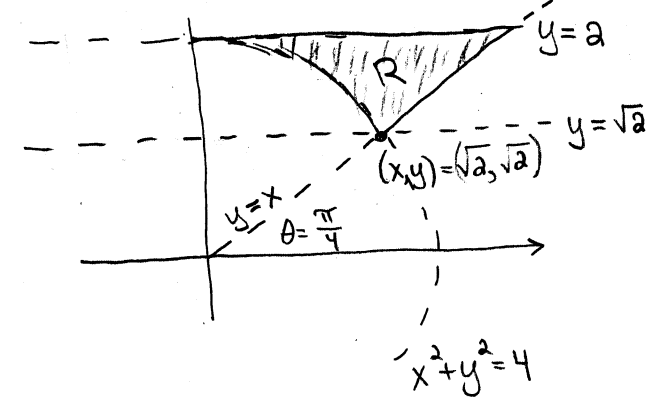
$$\text{Area} = \int_{\theta=\pi/4}^{\pi/3} \int_{r=\csc\theta}^{\sqrt{2}} r \, dr \, d\theta = \int_{\pi/4}^{\pi/3} \left(1 - \frac{1}{2} \csc^2 \theta\right) d\theta$$

$$= \theta + \frac{1}{2} \cot \theta \Big|_{\pi/4}^{\pi/3}$$

$$= \frac{\pi}{3} + \frac{1}{2\sqrt{3}} - \frac{\pi}{4} - \frac{1}{2} = \frac{\pi}{12} - \frac{3-\sqrt{3}}{6}$$

≈ 0.05047 UNITS²

2. (3 points) Convert to an equivalent integral in polar coordinates and evaluate.



$$\int_{\sqrt{2}}^2 \int_{\sqrt{4-y^2}}^y dx \, dy = \int_{\theta=\pi/4}^{\pi/2} \int_{r=2}^{2 \csc \theta} r \, dr \, d\theta$$

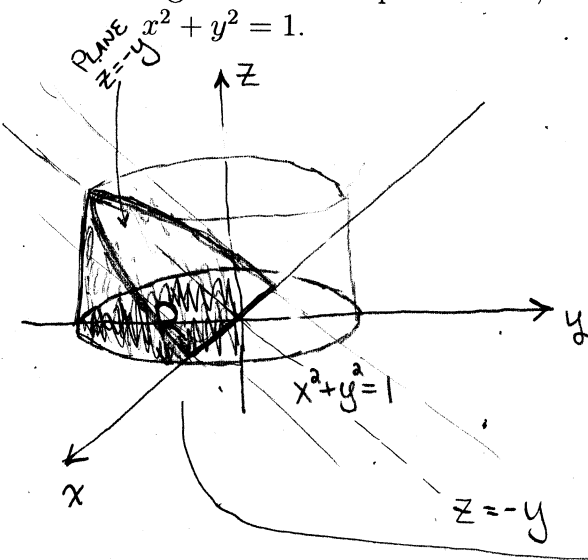
$$= \int_{\pi/4}^{\pi/2} (2 \csc^2 \theta - 2) d\theta = 2(-\cot \theta - \theta) \Big|_{\pi/4}^{\pi/2}$$

$$y=a \Rightarrow r=2 \csc \theta$$

$$= -2\left(0 + \frac{\pi}{2} - 1 - \frac{\pi}{4}\right) = 2 - \frac{\pi}{2}$$

≈ 0.4292

3. (4 points) Set up and evaluate the triple integral that gives the volume of the space region above the plane $z = 0$, below the plane $z = -y$, and inside the cylinder



$$\int_{\theta=\pi}^{2\pi} \int_{r=0}^1 \int_{z=0}^{-r \sin \theta} r \, dz \, dr \, d\theta$$

$$= \int_{\pi}^{2\pi} \int_0^1 -r^2 \sin \theta \, dr \, d\theta = \frac{1}{3} \int_{\pi}^{2\pi} -\sin \theta \, d\theta$$

$$= \frac{1}{3} (\cos \theta) \Big|_{\pi}^{2\pi} = \frac{1}{3} - \frac{1}{3}(-1) = \frac{2}{3}$$

$\frac{2}{3}$ UNITS³