Math 233 - Quiz 1

Name key

August 24, 2023

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due August 29.

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1. (1 point) The 2-dimensional vector $\vec{u}$ has magnitude 5 and makes a 225° with the positive x-axis. Write $\vec{u}$ in component form.

$$\vec{u} = 5\cos 225^\circ\, \hat{i} + 5\sin 225^\circ\, \hat{j}$$

$$= 5\left(-\frac{\sqrt{2}}{2}\right)\hat{i} + 5\left(-\frac{\sqrt{2}}{2}\right)\hat{j} \Rightarrow \boxed{\vec{u} = -\frac{5\sqrt{2}}{2}\hat{i} - \frac{5\sqrt{2}}{2}\hat{j}}$$

2. (4 points) Let $\vec{w}$ be the vector from $P(2,-1,-3)$ to $Q(-3,0,8)$.

   (a) Find the component form of $\vec{w}$.

$$\vec{w} = \overrightarrow{PQ} = (-3-2)\hat{i} + (0-(-1))\hat{j} + (8-(-3))\hat{k}$$

$$\boxed{\vec{w} = -5\hat{i} + \hat{j} + 11\hat{k}}$$

   (b) Compute $\|\vec{w}\|$.

$$\|\vec{w}\| = \sqrt{(-5)^2 + (1)^2 + (11)^2} = \sqrt{25+1+121} = \boxed{\sqrt{147} = 7\sqrt{3}}$$

   (c) Find a vector of magnitude 3 whose direction is opposite that of $\vec{w}$.

$$\frac{-3\vec{w}}{\|\vec{w}\|} = \frac{-3\vec{w}}{7\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{7}\vec{w} = \frac{\sqrt{3}}{7}\left(5\hat{i} - \hat{j} - 11\hat{k}\right)}$$

   (d) Compute $\|-5\vec{w}\|$.

$$\|-5\vec{w}\| = 5\|\vec{w}\| = \boxed{35\sqrt{3}}$$

Turn over.

3. (2 points) Determine whether the points $A(9,6,3)$, $B(-1,1,8)$, and $C(-11,-4,13)$ are collinear.

$\overrightarrow{AB} = -10\hat{\imath} - 5\hat{\jmath} + 5\hat{k}$

$\overrightarrow{AC} = -20\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$

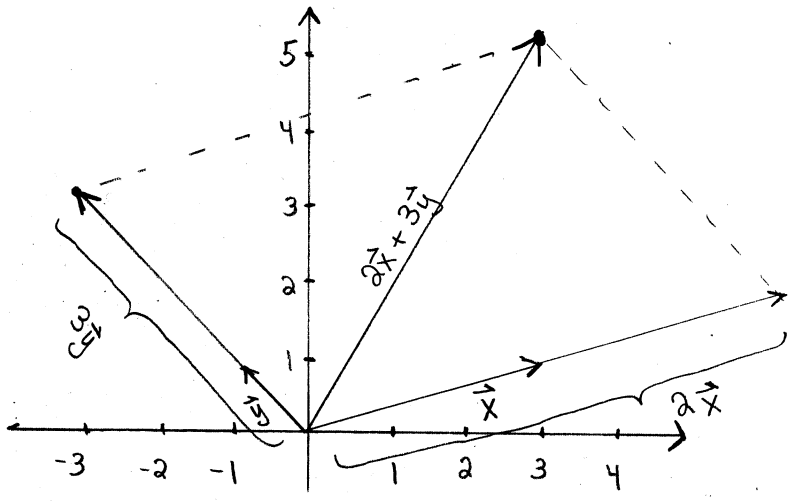
$\overrightarrow{AC} = 2\overrightarrow{AB} \Rightarrow$ $\overrightarrow{AC}$ AND $\overrightarrow{AB}$ ARE PARALLEL

$\Rightarrow$ A, B, C SHARE THE SAME LINE.

4. (3 points) Let $\vec{x} = 3\hat{\imath} + \hat{\jmath}$ and $\vec{y} = -\hat{\imath} + \hat{\jmath}$. Compute $\vec{z} = 2\vec{x} + 3\vec{y}$, and then sketch $\vec{x}$, $\vec{y}$, and $\vec{z}$, illustrating how they are related by the parallelogram law.

$\vec{z} = [6\hat{\imath} + 2\hat{\jmath}] + [-3\hat{\imath} + 3\hat{\jmath}]$

$\vec{z} = 3\hat{\imath} + 5\hat{\jmath}$



$\vec{z} = 2\vec{x} + 3\vec{y}$ OVERLAPS THE DIAGONAL OF THE PARALLELOGRAM DETERMINED BY $2\vec{x}$ AND $3\vec{y}$.