

Math 233 - Quiz 2

August 31, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) The first component of \vec{w} is 6. Find $\|\vec{w}\|$ if \vec{w} is parallel to $\langle -18, 9, -21 \rangle$.

$$k\vec{w} = \langle -18, 9, -21 \rangle$$

$$\vec{w} = \langle 6, -3, 7 \rangle$$

$$\vec{w} = \left\langle \frac{-18}{k}, \frac{9}{k}, \frac{-21}{k} \right\rangle$$

$$= \langle 6, _, _ \rangle$$

$$k = -3$$

$$\|\vec{w}\| = \sqrt{36 + 9 + 49}$$

$$= \sqrt{94}$$

2. (3 points) Let $\vec{u} = 3\hat{i} + \hat{j} + 5\hat{k}$ and $\vec{w} = -4\hat{i} + 2\hat{j} - 6\hat{k}$. Find the measure of the angle between \vec{u} and \vec{w} . Write your final answer in degrees rounded to the nearest tenth.

$$\vec{u} \cdot \vec{w} = \|\vec{u}\| \|\vec{w}\| \cos \theta$$

$$\vec{u} \cdot \vec{w} = -12 + 2 - 30 = -40$$

$$\cos \theta = \frac{-40}{\sqrt{35} \sqrt{56}}$$

$$\|\vec{u}\| = \sqrt{9 + 1 + 25} = \sqrt{35}$$

$$\|\vec{w}\| = \sqrt{16 + 4 + 36} = \sqrt{56}$$

$$\theta \approx 154.6^\circ$$

3. (3 points) Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + 3\hat{k}$. Find the projection of \vec{b} onto \vec{a} .

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a} = \frac{1}{6} (2\hat{i} - \hat{j} + \hat{k}) = \frac{1}{3}\hat{i} - \frac{1}{6}\hat{j} + \frac{1}{6}\hat{k}$$

$$\vec{a} \cdot \vec{b} = 0 - 2 + 3 = 1$$

$$\vec{a} \cdot \vec{a} = 4 + 1 + 1 = 6$$

4. (2 points) Find the cosine of the angle between $\vec{u} = 2\hat{i} - \hat{j} + \hat{k}$ and the positive x-axis.

$$\vec{u} \cdot \hat{i} = \|\vec{u}\| \|\hat{i}\| \cos \alpha$$

$$\vec{u} \cdot \hat{i} = 2 + 0 + 0 = 2$$

$$\|\vec{u}\| = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$\|\hat{i}\| = 1$$

$$\cos \alpha = \frac{\vec{u} \cdot \hat{i}}{\|\vec{u}\| \|\hat{i}\|}$$

$$= \frac{2}{\sqrt{6}}$$