

Math 233 - Quiz 3

September 7, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the area of $\triangle ABC$, where

$$A(1, -2, 4), \quad B(-2, 7, 3), \quad C(5, 5, -3).$$

$$\vec{AB} = -3\hat{i} + 9\hat{j} - \hat{k}$$

$$\vec{AC} = 4\hat{i} + 7\hat{j} - 7\hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 9 & -1 \\ 4 & 7 & -7 \end{vmatrix} = -56\hat{i} + 25\hat{j} - 57\hat{k}$$

$$\text{Area} = \frac{1}{2} \cdot \|\vec{AB} \times \vec{AC}\|$$

$$= \frac{1}{2} \sqrt{56^2 + 25^2 + 57^2}$$

$$= \frac{1}{2} \sqrt{7010} \approx 41.86 \text{ units}^2$$

2. (3 points) Find a unit vector that is orthogonal to both $\vec{u} = 3\hat{i} + \hat{j} + 5\hat{k}$ and $\vec{w} = -4\hat{i} + 2\hat{j} - 6\hat{k}$.

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 5 \\ -4 & 2 & -6 \end{vmatrix}$$

$$= -16\hat{i} - 2\hat{j} + 10\hat{k}$$

$$\|\vec{u} \times \vec{w}\| = \sqrt{16^2 + 2^2 + 10^2} = \sqrt{360} = 6\sqrt{10}$$

$$\frac{\vec{u} \times \vec{w}}{6\sqrt{10}} = \frac{-1}{3\sqrt{10}} (8\hat{i} + \hat{j} - 5\hat{k})$$

3. (3 points) Find parametric and symmetric equations for the line passing through $(1, -3, 5)$ and $(8, 9, -2)$.

P Q

$$\vec{v} = \vec{PQ} = 7\hat{i} + 12\hat{j} - 7\hat{k}$$

Using initial point P...

$$x = 7t + 1$$

$$y = 12t - 3$$

$$z = -7t + 5$$

PARAMETRIC

$$\frac{x-1}{7} = \frac{y+3}{12} = \frac{z-5}{-7}$$

SYMMETRIC

4. (1 point) What does it mean for two vectors to be orthogonal?

THEIR DOT PRODUCT IS ZERO.