Math 233 - Quiz 4
September 21, 2023

Name key

Score ________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 26.

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1. (2 points) Find $\vec{r}(t)$ if $\vec{r}\,'(t) = -8\cos(4t)\,\hat{\imath} + te^{-t}\,\hat{\jmath} + \dfrac{3}{t^2+1}\,\hat{k}$ and $\vec{r}(0) = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$.

$$\vec{r}(t) = \int -8\cos 4t \, dt \; \hat{\imath} + \underbrace{\int te^{-t} dt}_{\text{PARTS}} \; \hat{\jmath} + \int \frac{3}{t^2+1} dt \; \hat{k}$$

+	t	e^{-t}
−	1	$-e^{-t}$
+	0	e^{-t}

$$= \left(-2\sin 4t + c_1\right)\hat{\imath} + \left(-te^{-t} - e^{-t} + c_2\right)\hat{\jmath} + \left(3\tan^{-1} t + c_3\right)\hat{k}$$

$$\vec{r}(0) = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k} = c_1\hat{\imath} + (-1 + c_2)\hat{\jmath} + c_3\hat{k} \Rightarrow c_1 = 4,\; c_2 = 4,\; c_3 = -2$$

$$\vec{r}(t) = \left(-2\sin 4t + 4\right)\hat{\imath} + \left(-te^{-t} - e^{-t} + 4\right)\hat{\jmath} + \left(3\tan^{-1} t - 2\right)\hat{k}$$

2. (2 points) For $t > 0$, let $\vec{r}(t) = (\cos t + t\sin t)\hat{\imath} + (\sin t - t\cos t)\hat{\jmath}$. Compute the unit tangent vector, $\hat{T}(t)$.

$$\vec{r}\,'(t) = \left(-\sin t + \sin t + t\cos t\right)\hat{\imath} + \left(\cos t - \cos t + t\sin t\right)\hat{\jmath}$$

$$= t\cos t \; \hat{\imath} + t\sin t \; \hat{\jmath}$$

$$\|\vec{r}\,'(t)\| = \sqrt{t^2\cos^2 t + t^2\sin^2 t} = \sqrt{t^2} = t, \; t > 0$$

$$\hat{T}(t) = \cos t \; \hat{\imath} + \sin t \; \hat{\jmath}$$

Turn over.

3. (2 points) Find a vector-valued function whose graph is the line <u>segment</u> from $(1,-3,5)$ and $(8,9,-2)$.

$\vec{u} = 7\hat{i} + 12\hat{j} - 7\hat{k}$ Using $(1,-3,5)$, we get parametric equations

$$x = 7t+1$$
$$y = 12t-3 \quad 0 \le t \le 1$$
$$z = -7t+5$$

$$\boxed{\vec{r}(t) = (7t+1)\hat{i} + (12t-3)\hat{j} + (-7t+5)\hat{k}, \quad 0 \le t \le 1}$$

4. (2 points) An object starts from rest at the point $P(1,2,0)$ and moves with acceleration $\vec{a}(t) = \hat{j} + 2\hat{k}$, where distances are measured in feet and time in seconds. Find the location of the object after 2 seconds.

$\vec{a}(t) = \hat{j} + 2\hat{k} \Rightarrow \vec{v}(t) = c_1\hat{i} + (t+c_2)\hat{j} + (2t+c_3)\hat{k}$

$\vec{v}(0) = \vec{0} \Rightarrow c_1 = 0, c_2 = 0, c_3 = 0$

$\vec{v}(t) = t\hat{j} + 2t\hat{k} \Rightarrow \vec{r}(t) = c_1\hat{i} + (\frac{1}{2}t^2 + c_2)\hat{j} + (t^2 + c_3)\hat{k}$

$\vec{r}(0) = 1\hat{i} + 2\hat{j} + 0\hat{k}$

$\Rightarrow c_1 = 1, c_2 = 2, c_3 = 0$

$\vec{r}(2) = \hat{i} + 4\hat{j} + 4\hat{k}$ $\longleftarrow$ $\vec{r}(t) = \hat{i} + (\frac{1}{2}t^2 + 2)\hat{j} + t^2\hat{k}$

$\boxed{(1,4,4)}$

5. (2 points) Let $\vec{r}(t) = (t^2-t)\,\hat{\imath} + \frac{1}{6}(4t-1)^{3/2}\,\hat{\jmath} + 5\,\hat{k}$. Starting from $t=1$, compute the arc-length parameter s.

$$\vec{r}\,'(t) = (2t-1)\hat{i} + \frac{1}{4}(4t-1)^{1/2}(4)\hat{j} = (2t-1)\hat{i} + \sqrt{4t-1}\,\hat{j}$$

$$\|\vec{r}\,'(t)\| = \sqrt{(2t-1)^2 + 4t-1} = \sqrt{4t^2} = 2t, \quad t \ge 1$$

$$s = \int_1^t 2u\,du = u^2\Big|_1^t = t^2 - 1$$

$$\boxed{s = t^2 - 1, \quad t \ge 1}$$