

**Math 233 - Quiz 5**

September 28, 2023

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Let  $\vec{r}(t) = t^2\hat{i} + t^3\hat{j} + t\hat{k}$ . Compute  $\hat{T}(t)$ .

$$\vec{r}'(t) = 2t\hat{i} + 3t^2\hat{j} + \hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{4t^2 + 9t^4 + 1}$$

$$\hat{T}(t) = \frac{2t\hat{i} + 3t^2\hat{j} + \hat{k}}{\sqrt{1 + 4t^2 + 9t^4}}$$

2. (3 points) What are the three important characteristics of the unit tangent vector?

① IT IS A UNIT VECTOR (MAGNITUDE 1).

② IT IS TANGENT TO THE CURVE.

③ IT POINTS IN THE DIRECTION OF MOTION (OR ORIENTATION).

3. (4 points) Let  $\vec{r}(t) = (4t^2 + 3)\hat{i} + 5\hat{j} + (t^2 + 7)\hat{k}$ . Reparameterize  $r$  in terms of the arc length parameter starting from  $t = 0$ .

$$\vec{r}'(t) = 8t\hat{i} + 2t\hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{64t^2 + 4t^2} = \sqrt{68}t, \quad t \geq 0$$

$$s = \int_0^t \sqrt{68} u \, du = \frac{\sqrt{68}}{2} t^2 = \sqrt{17} t^2$$

$$\vec{R}(s) = \left(\frac{4s}{\sqrt{17}} + 3\right)\hat{i} + 5\hat{j} + \left(\frac{s}{\sqrt{17}} + 7\right)\hat{k}$$

$$s = \sqrt{17} t^2 \Rightarrow t^2 = \frac{s}{\sqrt{17}}$$

4. (1 point) Let  $\vec{R}(s)$  be your reparameterization above. What is the magnitude of  $\vec{R}'(s)$ ?

$$\|\vec{R}'(s)\| = 1$$

I DON'T NEED TO COMPUTE IT;

WE HAVE A THEOREM THAT SAYS

IT MUST BE SO!