Math 233 - Quiz 7

October 19, 2023

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 24.

1. (4 points) Consider the following limit: $\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$.

   (a) Show that the limit does not exist.

ALONG $y=0$:

$$\lim_{x\to 0} \frac{0}{x^2} = \boxed{0}$$

ALONG $y=x$:

$$\lim_{x\to 0} \frac{x^2+x^3}{2x^2} = \lim_{x\to 0} \frac{1+x}{2} = \boxed{\frac{1}{2}}$$

TWO DIFF. LIMITS ALONG TWO PATHS $\Downarrow$ LIMIT DNE.

   (b) Now think about this new limit: $\lim_{(x,y)\to(1,0)} \frac{(x-1)y+y^3}{(x-1)^2+y^2}$.
   This limit also does not exist. What paths could you use to prove it?

$\boxed{y=0}$ AND $\boxed{x=y+1}$

$$\lim_{x\to 0} \frac{0}{x^2} = 0 \qquad \lim_{y\to 0} \frac{y^2+y^3}{2y^2} = \frac{1}{2}$$

THESE ARE THE SAME PATHS AS ABOVE ACCOUNTING FOR THE SHIFT 1 UNIT TO THE RIGHT

2. (2 points) Let $z = \ln(xy+y^2)$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial x} = \frac{y}{xy+y^2} \qquad \frac{\partial z}{\partial y} = \frac{x+2y}{xy+y^2}$$

Turn over.

3. (2 points) Show that there is no number k for which the following function is continuous at $(0,0)$.

$$f(x,y) = \begin{cases} \dfrac{x^4 - 4y^2}{x^2+2y^2}, & (x,y) \neq (0,0) \\ k, & (x,y) = (0,0) \end{cases}$$

LET'S LOOK AT

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}.$$

IF LIMIT EXISTS, THEN k MUST BE THAT LIMIT.

ALONG $x = 0$:

$$\lim_{y\to 0} \frac{-4y^2}{2y^2} = -2$$

ALONG $y = 0$:

$$\lim_{x\to 0} \frac{x^4}{x^2} = \lim_{x\to 0} x^2 = 0$$

$\Rightarrow$ $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE.

No SUCH k EXISTS!

4. (2 points) Compute the limit: $\displaystyle\lim_{(x,y)\to(3,3)} \frac{(2x+y)^2 - (5y^2+4xy)}{x-y}$

%

$$\lim_{(x,y)\to(3,3)} \frac{4x^2 + \cancel{4xy} + y^2 - 5y^2 - \cancel{4xy}}{x-y}$$

$$= \lim_{(x,y)\to(3,3)} \frac{4x^2 - 4y^2}{x-y}$$

$$= \lim_{(x,y)\to(3,3)} \frac{4(x+y)\cancel{(x-y)}}{\cancel{x-y}} = \boxed{24}$$