

Math 233 - Quiz 7

October 19, 2023

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due October 24.

1. (4 points) Consider the following limit: $\lim_{(x,y) \rightarrow (0,0)} \frac{xy + y^3}{x^2 + y^2}$.

(a) Show that the limit does not exist.

(b) Now think about this new limit: $\lim_{(x,y) \rightarrow (1,0)} \frac{(x-1)y + y^3}{(x-1)^2 + y^2}$.

This limit also does not exist. What paths could you use to prove it?

2. (2 points) Let $z = \ln(xy + y^2)$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Turn over.

3. (2 points) Show that there is no number k for which the following function is continuous at $(0, 0)$.

$$f(x, y) = \begin{cases} \frac{x^4 - 4y^2}{x^2 + 2y^2}, & (x, y) \neq (0, 0) \\ k, & (x, y) = (0, 0) \end{cases}$$

4. (2 points) Compute the limit: $\lim_{(x,y) \rightarrow (3,3)} \frac{(2x + y)^2 - (5y^2 + 4xy)}{x - y}$